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Longitudinal diagnostic models for small sample size contexts

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Cognitive diagnosis modelling

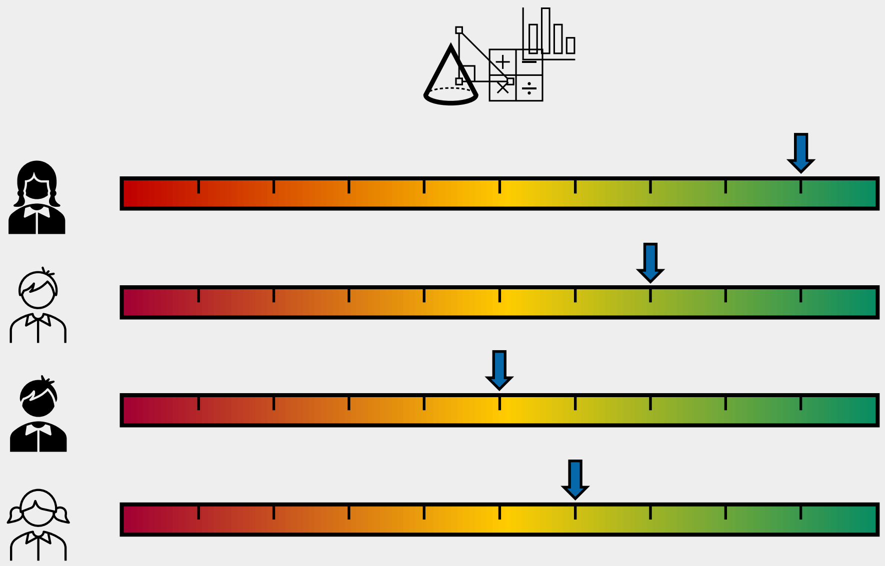
Cognitive diagnosis modelling (**CDM**) is a family of confirmatory statistical models that can be understood either as:

- restricted latent class models, or
- multidimensional item response theory (IRT) models with discrete latent variables

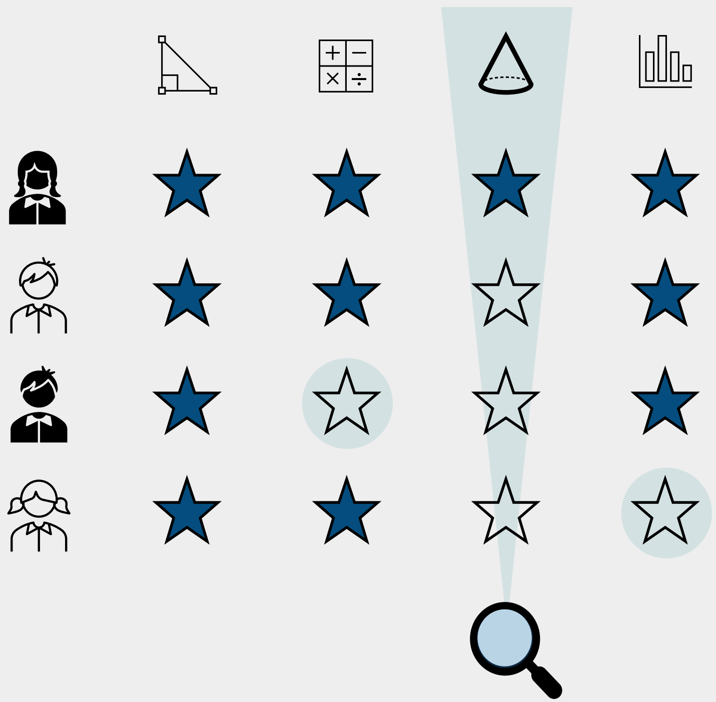
These models have been primarily employed in educational assessments, where the detailed feedback they provide can help identify students' strengths and weaknesses objectively and in a timely manner (de la Torre & Minchen, 2014; Paulsen & Svetina, 2022)

Cognitive diagnosis modelling

Summative assessments



Diagnostic assessments



Longitudinal CDM

In the last decade, several longitudinal CDMs have been developed with the aim of evaluating changes in respondents' attribute profiles over time

The calibrate-and-score (**CS**; Jurich & Bradshaw, 2014) method involves:

1. Fitting a CDM to the pre-test data

   Calibrating items and obtaining attribute profile classifications

2. Fitting a CDM to the post-test data, constraining the item parameters to the pre-test estimates

   Obtaining attribute profile classifications

This approach does not account for the interdependency at pre- and post-test

Estimation is simplified, with only $2^K - 1$ parameters estimated at post-test

Longitudinal CDM

The transition diagnostic classification model (**TDCM**; Madison & Bradshaw, 2018) is a latent transition analysis (LTA) with a CDM as the measurement model

It estimates three groups of parameters concurrently for pre- and post-test data:

- The probabilities of belonging to each latent class at pre-test
- The probabilities of transitioning from one latent class to another across time
- Item response probabilities at pre- and post-test (via CDM)

This model accounts for local dependency between responses

It enables longitudinal invariance assessment

Estimation is challenging and computationally expensive, with a total of $\sum_{j=1}^{J} 2^{K_j} + 2(2^K - 1) + 2^K(2^K - 1)$ parameters estimated concurrently

Model complexity and sample size

CDM is a family of models with different item response functions and complexities

In order to provide accurate parameter estimates, traditional CDMs require:

- Large sample sizes (*N* > 500; Sen & Cohen, 2021)
- Simple Q-matrices, high-quality items, low dimensionality… (Sorrel et al., 2021)

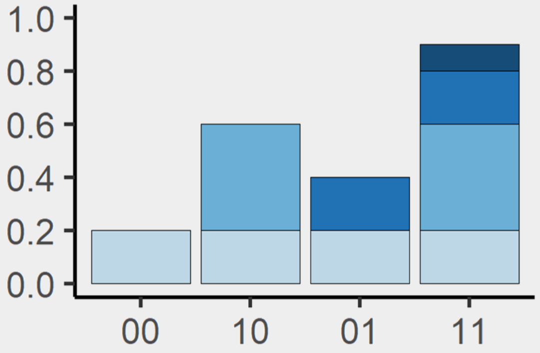
Under suboptimal conditions, a more parsimonious model might provide more accurate parameter estimates and attribute classifications than a more complex one, even when the latter is the generating model (Nájera et al., 2023)

Model complexity and sample size

CDM is a family of models with different item response functions and complexities

G-DINA

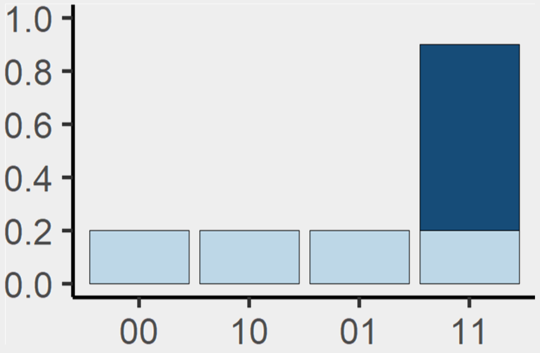
(de la Torre, 2011)



$$(2^K - 1) + \sum_{j=1}^{J} \mathbf{2}^{\boldsymbol{K_j}}$$

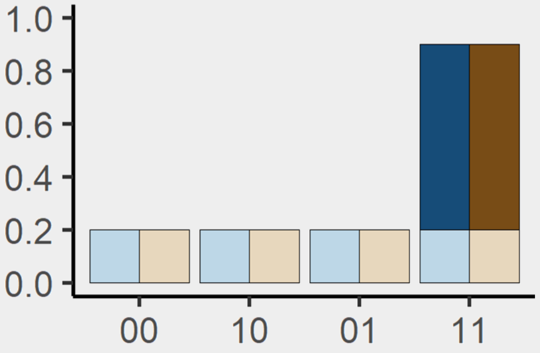
DINA

(Junker & Sijtsma, 2001)



$$(2^K - 1) + \boldsymbol{2J}$$

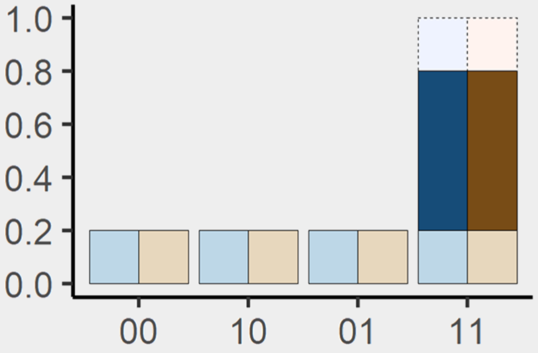
S-DINA



$$(2^K - 1) + \mathbf{2}$$

R-DINA

(Nájera et al., 2023)



$$(2^K - 1) + \mathbf{1}$$

The present study

Up to date, studies about longitudinal CDM have worked with relatively optimal contexts (e.g., simple Q-matrices) and large sample sizes ($N > 1000$)

The main objective of the present study is to elucidate the most optimal way of estimating longitudinal CDM in small sample size scenarios

- Is the simplicity of the **CS** method enough to overcome its limitations and outperform the **TDCM**?
- Can simpler models (e.g., **R-DINA**) provide more accurate parameter estimates and attribute classifications than the **G-DINA** model, even when the G-DINA is the true, generating model?

Simulation study design

Simulation factors

Generating model = **G-DINA**

Sample size = 25, 50, 100

Number of attributes = 3, 5

Q-matrix complexity = simple, complex (.45, .35, .20)

Item discrimination = 0.1, 0.2, 0.3

Probability of attribute mastery growth = 0.20

Probability of attribute mastery loss = 0.10

Item parameter drift = none, random

6 items per attribute, attribute correlations of 0.5

Methods

TDCM CS

×

G-DINA DINA S-DINA R-DINA

Dependent variables

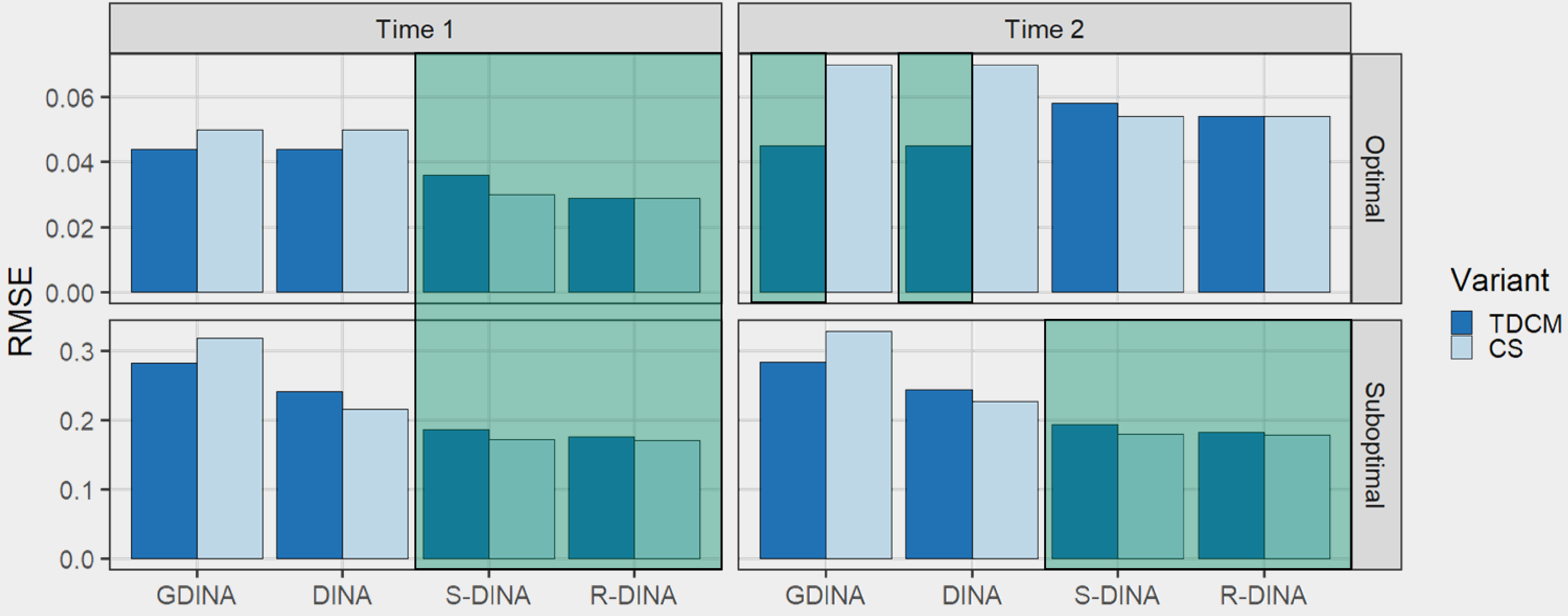
Item parameter recovery

Transition probs. Recovery

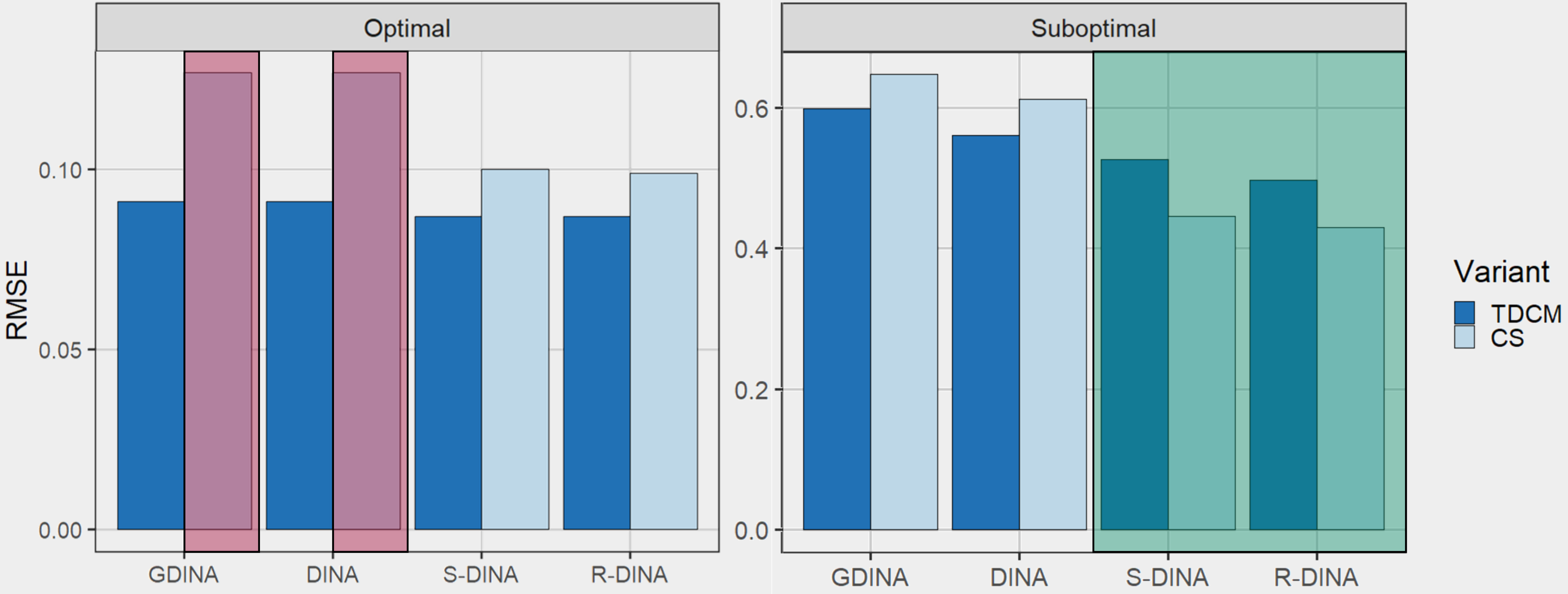
Classification accuracy

Model fit

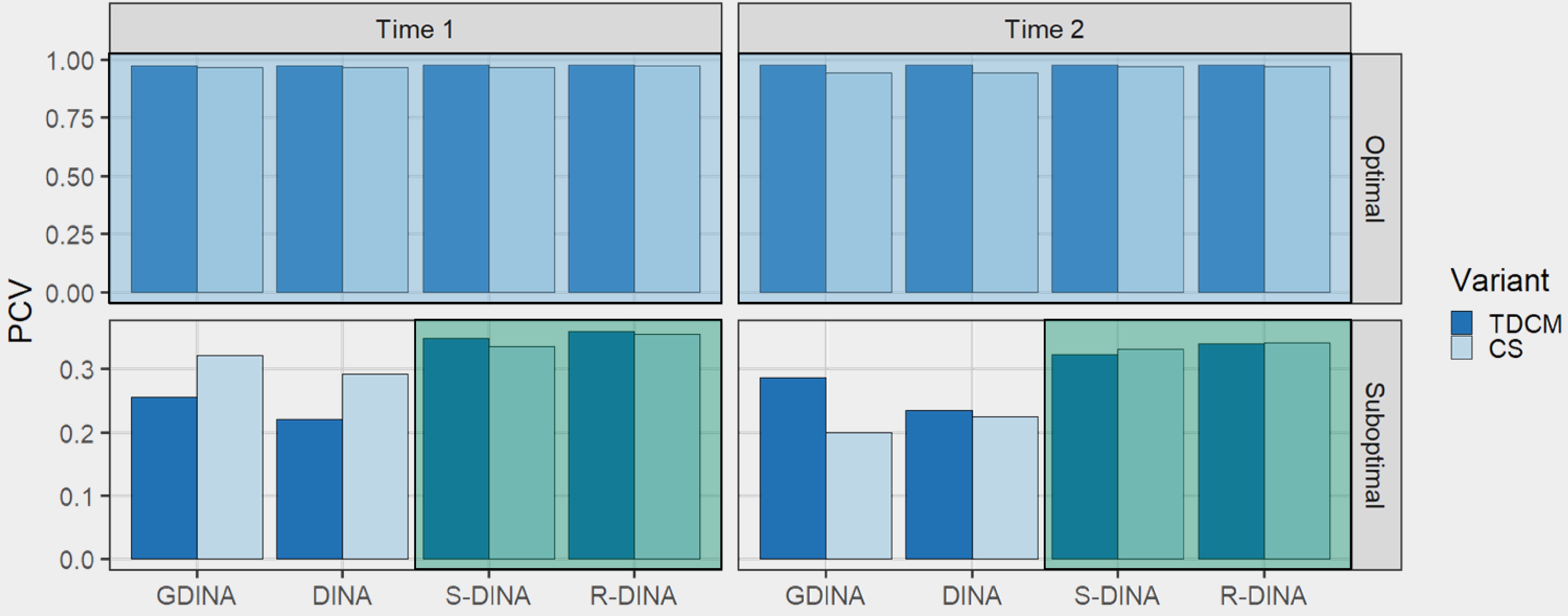
Results (I): Item parameter recovery



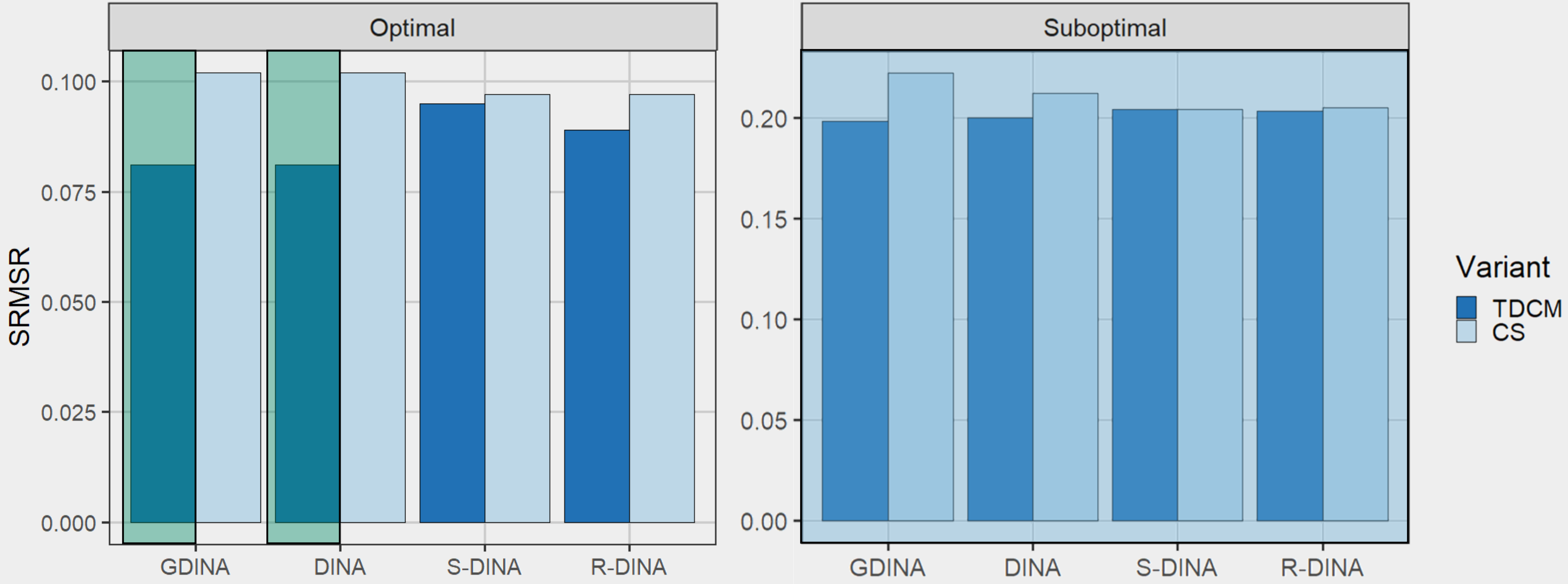
Results (II): Transition probabilities recovery



Results (III): Attribute classification accuracy



Results (IV): Absolute fit



Conclusions

More accurate **parameter estimates** and **attribute profile classifications** are provided by **simple CDMs** (S-DINA and R-DINA), especially under suboptimal conditions

Model fit is similar for **all CDMs**

The **TDCM** provides slightly better results with **complex CDMs** (G-DINA and DINA), especially under optimal conditions

The **CS** method provides slightly better results with **simple CDMs** (S-DINA and R-DINA), especially under suboptimal conditions

Overall…

1. S-DINA + CS
2. R-DINA + CS/TDCM
3. G-DINA + TDCM

Conclusions

Future extensions:

1. Anchor item design (e.g., different items on pre- and post-test)

   CDM classification invariance (Madison & Bradshaw, 2018)

2. Different modelling approaches of latent class transition

   Multilevel CDM (Huang, 2017)

   Hidden Markov models (Wang et al., 2018)

   Bayesian networks (Lee & Gu, 2024)

Conclusions

Research on CDM for small sample sizes

- Model comparison to reduce complexity (Sorrel et al., 2021)
- Reliability estimation via multiple imputation (Kreitchmann et al., 2023)
- Estimation approaches for small samples (Sorrel et al., 2023; Nájera et al., 2023)
- Computerized adaptive testing with no calibration sample (Nájera et al., 2025)

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cdmTools cdcatR cdcatS


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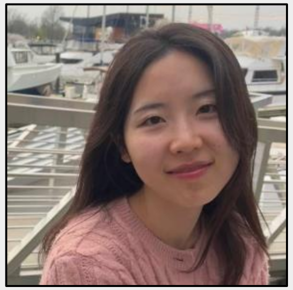
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Thank you!

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