EAM2025

XI Conference

23RD - 25TH JULY 2025

Spain Tenerife
Canary Islands

European Association of Methodology

Comparing nested multilevel models with the Vuong test

José María Santa Olalla Tovar, José Ángel Martínez-Huertas, Belén Fernández Castilla

Universidad de La Laguna
Cabildo de Tenerife
Tagoro
cajasiete
Gobierno de Canarias
Consejería de Universidades, Ciencia e Innovación y Cultura
Agencia Canaria de Investigación, Innovación y Sociedad de la Información
ICI Instituto Canario de Igualdad
tea
hogrefe

Vuong, Q. H. (1989). Likelihood ratio tests for model selection and non-nested hypotheses.

***Econometrica: Journal of the Econometric Society*, 307-333. DOI:10.2307/1912557**

Vuong determine the p.d.f. of the Likelihood Ratio statistic for nested, non nested, fully specified and misspecified models

There is no difference between
Vuong test and
Likelihood Ratio Test (LRT) for nested lineal models

Vuong test is implemented for SEM and MIXED MODELS in nonnest2 R package by Merkle and You

EMPIRICAL EXAMPLE

CASTILLA Y LEÓN STUDENTS

PISA 2022 DATABASE

Teacher Support

(PREDICTORS)

If student ASKs when he/she does not understand

Mathematics Anxiety

(DEPENDENT VARIABLE)

NESTED MODELS

$$ANXMAT_{ij} = \beta_{00} + U_j + e_{ij}$$

$$U_j \sim N(0, \tau_{00}^2) \quad e_{ij} \sim N(0, \sigma^2)$$

MODEL 0

$$ANXMAT_{ij} = \beta_{00} + \beta_1 \cdot TEACHSUP_{ij} + U_j + e_{ij}$$

$$U_j \sim N(0, \tau_{00}^2) \quad e_{ij} \sim N(0, \sigma^2)$$

MODEL 1

$$ANXMAT_{ij} = \beta_{00} + \beta_1 \cdot TEACHSUP_{ij} + \beta_2 \cdot ASK_{ij} + U_j + e_{ij}$$

$$U_j \sim N(0, \tau_{00}^2) \quad e_{ij} \sim N(0, \sigma^2)$$

MODEL 2

$$\beta_1 = 0$$
$$\beta_2 = 0$$

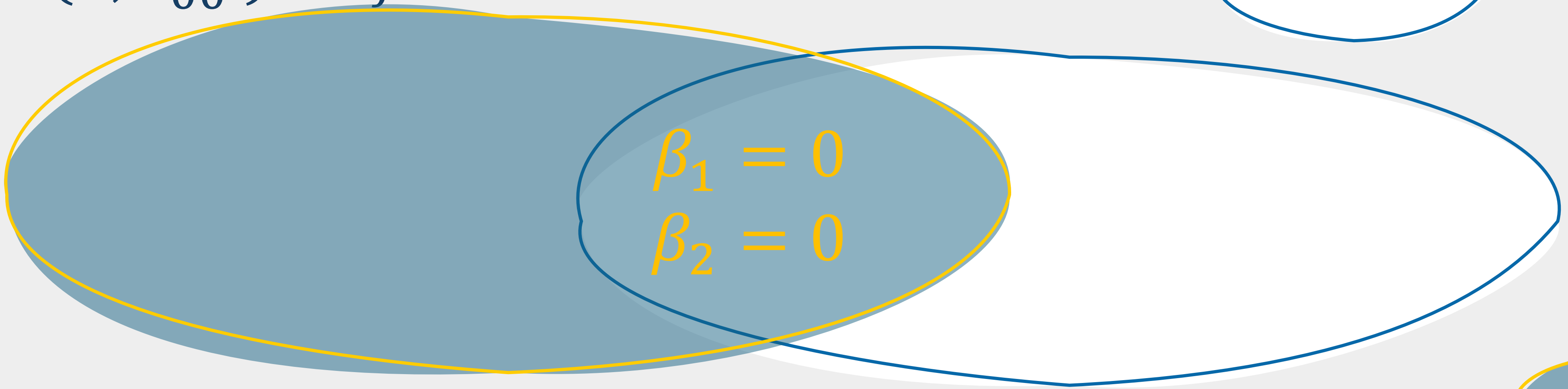
$$\beta_2 = 0$$

OVERLAPPING MODEL (NON NESTED)

$$ANXMAT_{ij} = \beta_{00} + \beta_1 \cdot TEACHSUP_{ij} + U_j + e_{ij}$$

$$U_j \sim \mathrm{N}(0, \tau_{00}^2) \quad e_{ij} \sim \mathrm{N}(0, \sigma^2)$$

MODEL 1



MODEL 3

$$ANXMAT_{ij} = \beta_{00} + \beta_2 \cdot ASK_{IJ} + U_j + e_{ij}$$

$$U_j \sim \mathrm{N}(0, \tau_{00}^2) \quad e_{ij} \sim \mathrm{N}(0, \sigma^2)$$

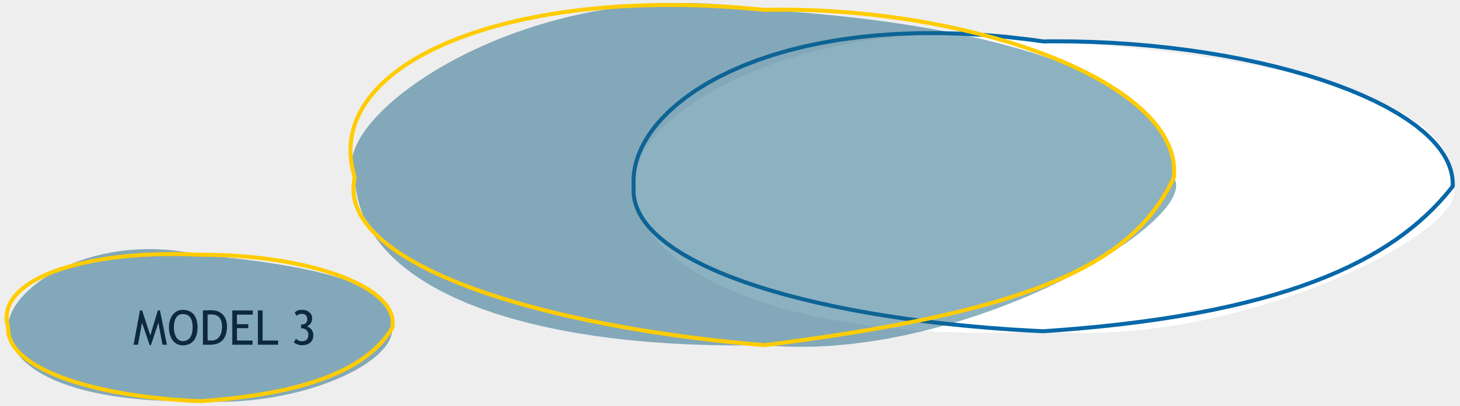
OVERLAPING MODEL (NON NESTED)

1) First, A test is applied to check if the models are distinguishable.
2) If they are, another test is applied to select the best model.

$$ANXMAT_{ij} = \beta_{00} + \beta_1 \cdot TEACHSUP_{ij} + U_j + e_{ij}$$

MODEL 1

$$U_j \sim N(0, \tau_{00}^2) \quad e_{ij} \sim N(0, \sigma^2)$$



VUONG TEST RESULTS,
1)MODELS ARE DISTINGUISHABLE, p = 0.002
2) MODEL 1 FITS BETTER THAN MODEL 3, p = 0.037

$$ANXMAT_{ij} = \beta_{00} + \beta_2 \cdot ASK_{IJ} + U_j + e_{ij}$$

$$U_j \sim N(0, \tau_{00}^2) \quad e_{ij} \sim N(0, \sigma^2)$$

OVERLAPPING MODEL (NON NESTED)

A prerequisite for Vuong test is sample independence.
In nonnest2, Vuong test is applied to level 2 units, because those are independent.

Vuong test allows comparing nested and non nested models, but in the case of nested mixed models, could the Merkle and You implementation in nonnest2 R package outperform LRT?

Simulation MLM

Level 1 Units N1	Level 2 Units N2	ICC	Standardized size effect. β	Total Variance
15	50	.1	0	0.47
30	150	.25	.05	1.41
		.50	.15	
			.25	

Simulation MLM

MODEL 0

$$Y_{ij} = \beta_{00} + U_j + e_{ij}$$

$$U_j \sim N(0, \tau_{00}^2) \quad e_{ij} \sim N(0, \sigma^2)$$

MODEL 1

$$Y_{ij} = \beta_{00} + \beta_1 \cdot X_{ij}^1 + U_j + e_{ij}$$

$$U_j \sim N(0, \tau_{00}^2) \quad e_{ij} \sim N(0, \sigma^2)$$

MODEL 2

$$Y_{ij} = \beta_{00} + \beta_1 \cdot X_{ij}^1 + \beta_2 \cdot X_{ij}^2 + U_j + e_{ij}$$

$$U_j \sim N(0, \tau_{00}^2) \quad e_{ij} \sim N(0, \sigma^2)$$

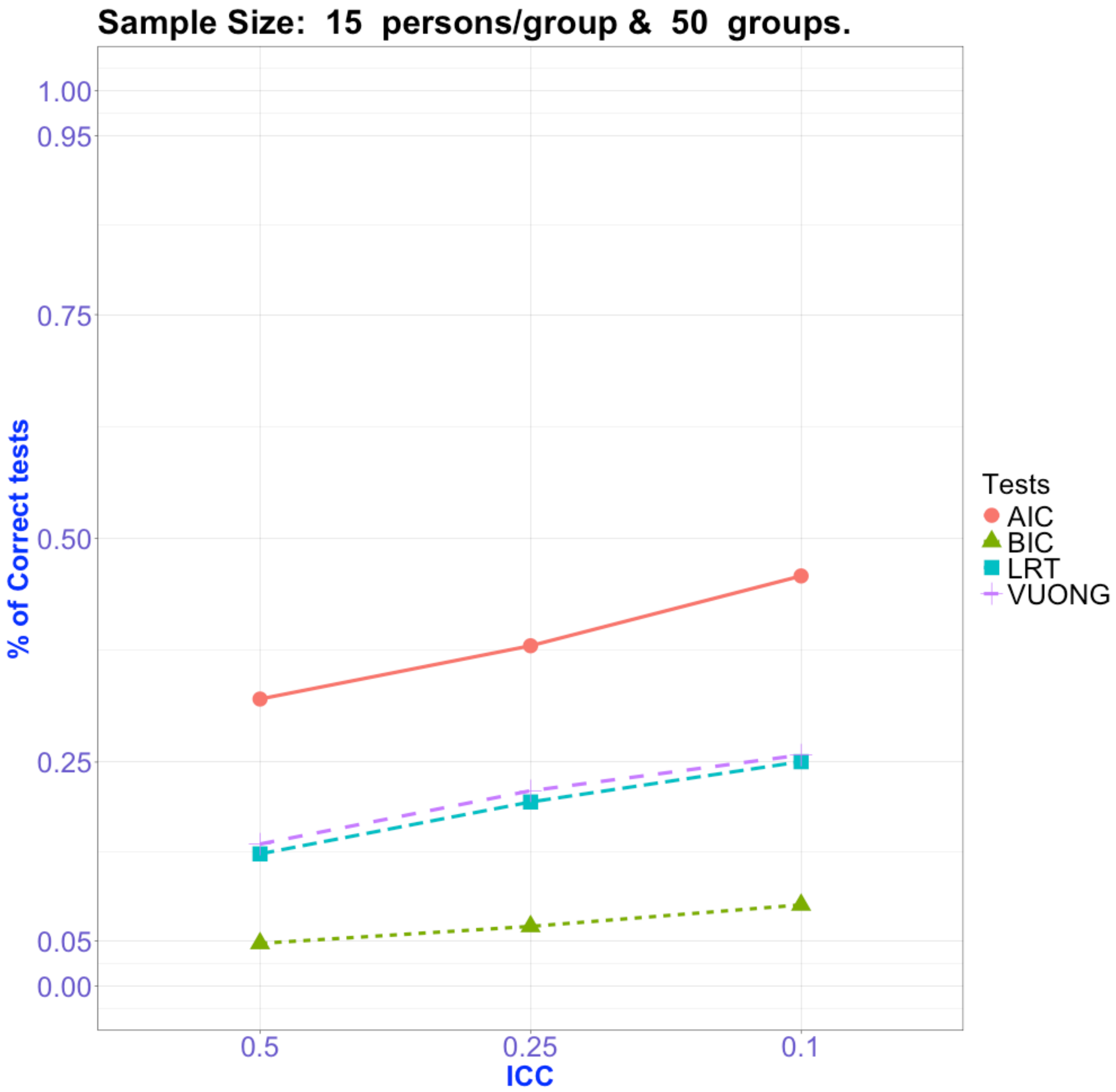
X_{ij}^2 is a variable which is not included in the data

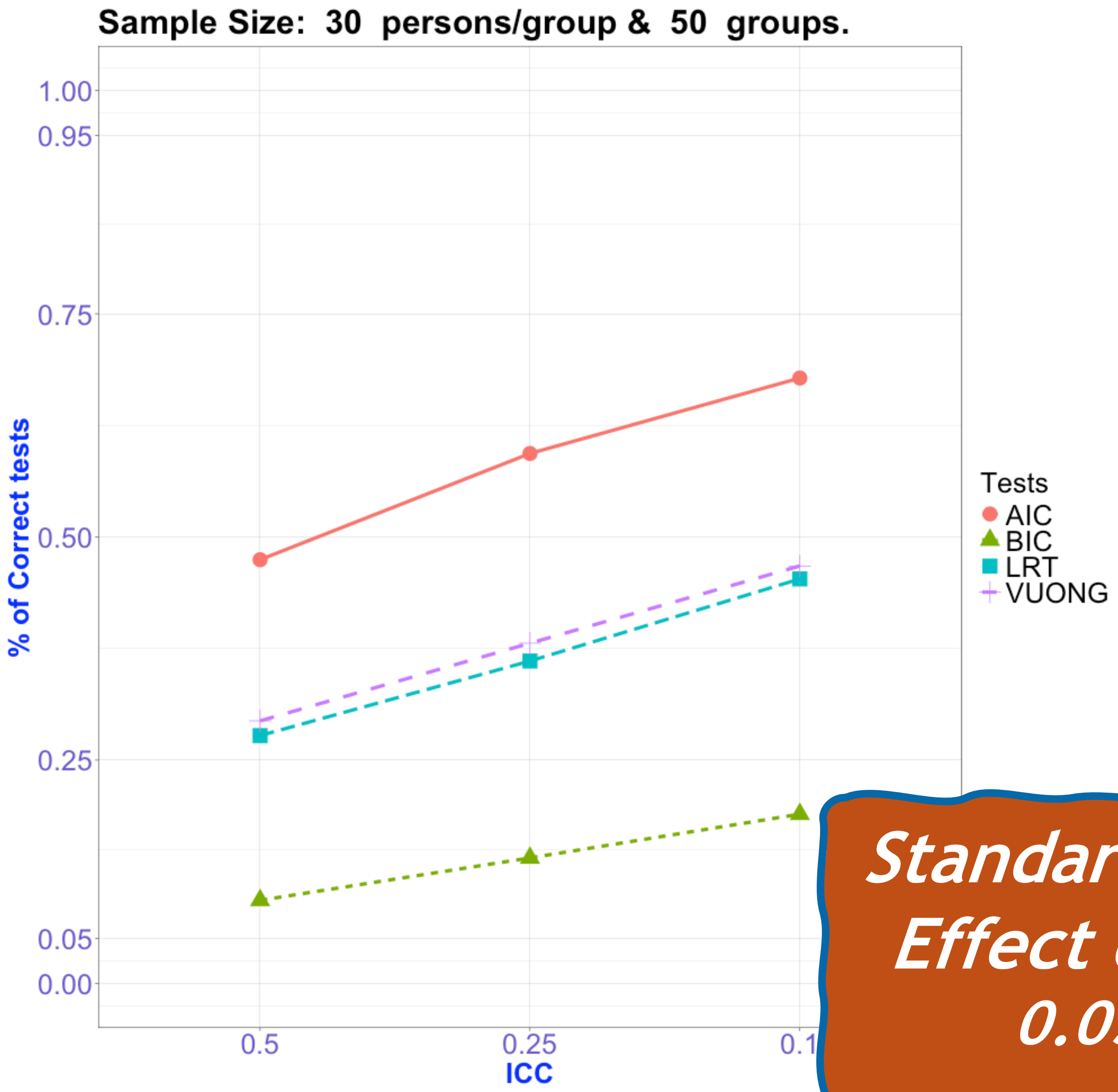
EAM2025
XI Conference

European Association of Methodology
Spain Tenerife Canary Islands

23RD - 25TH JULY 2025

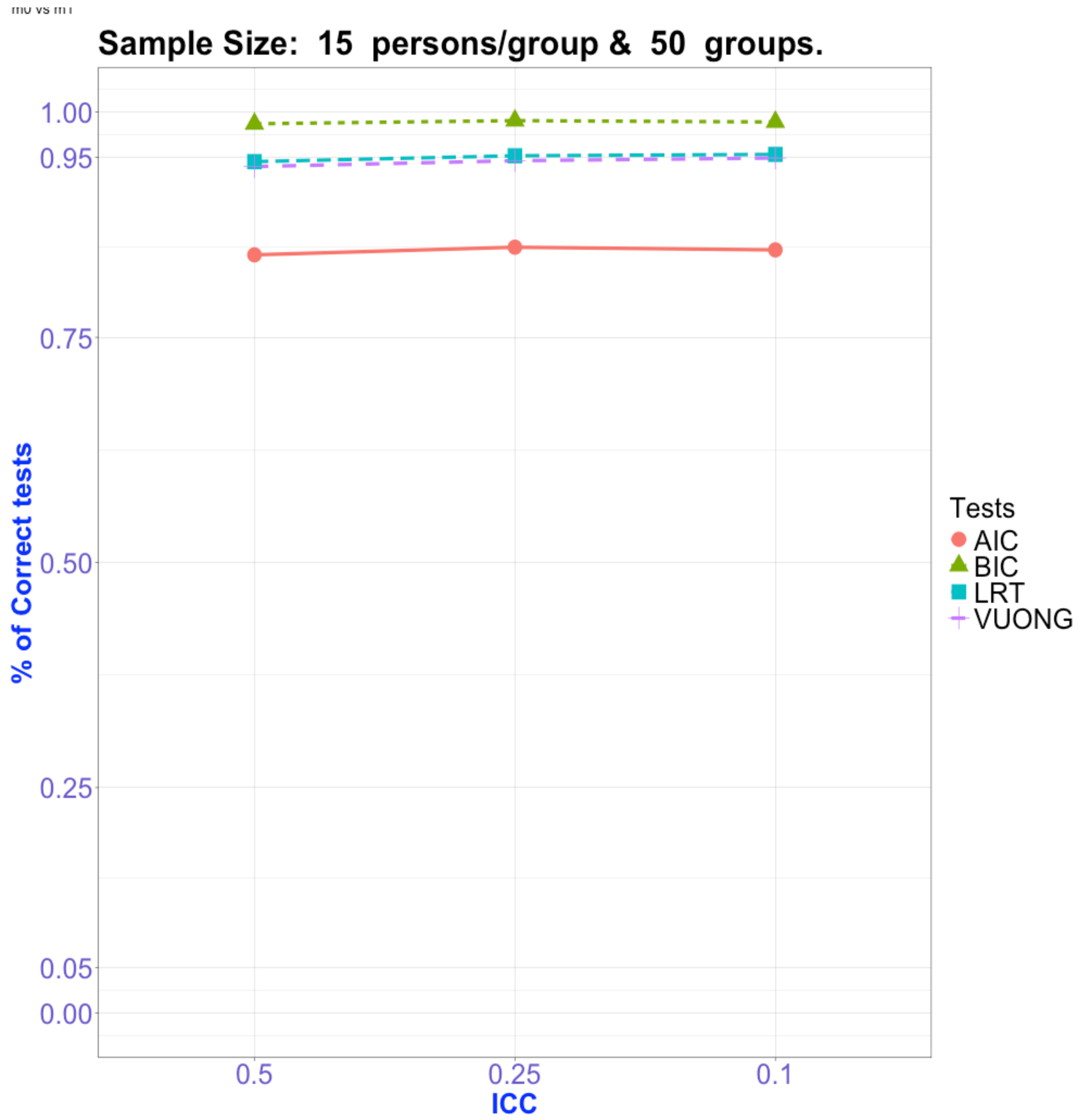
m0 vs m1

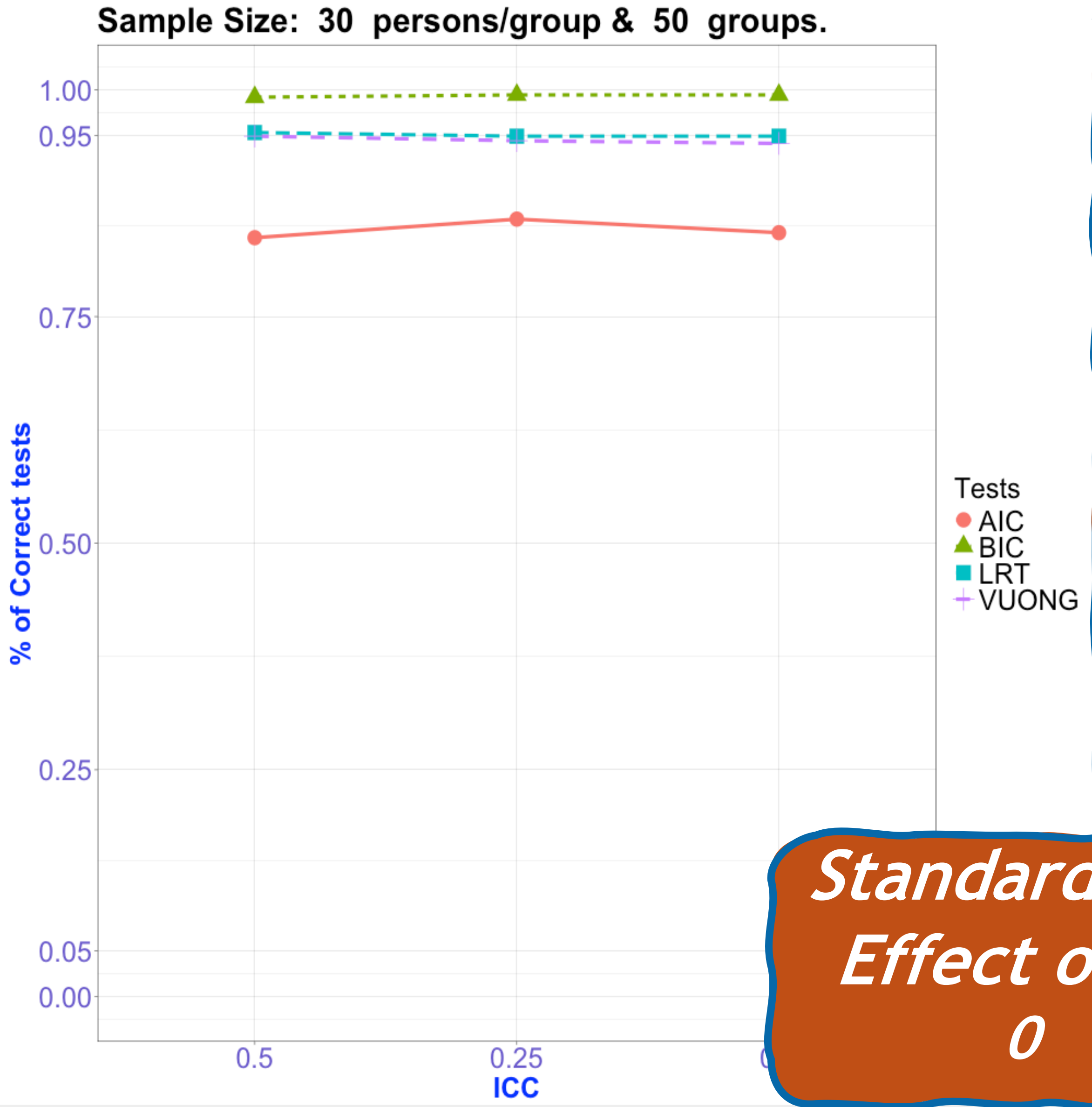




MODEL 0 VS MODEL 1

Standardized Effect of X1 0.05

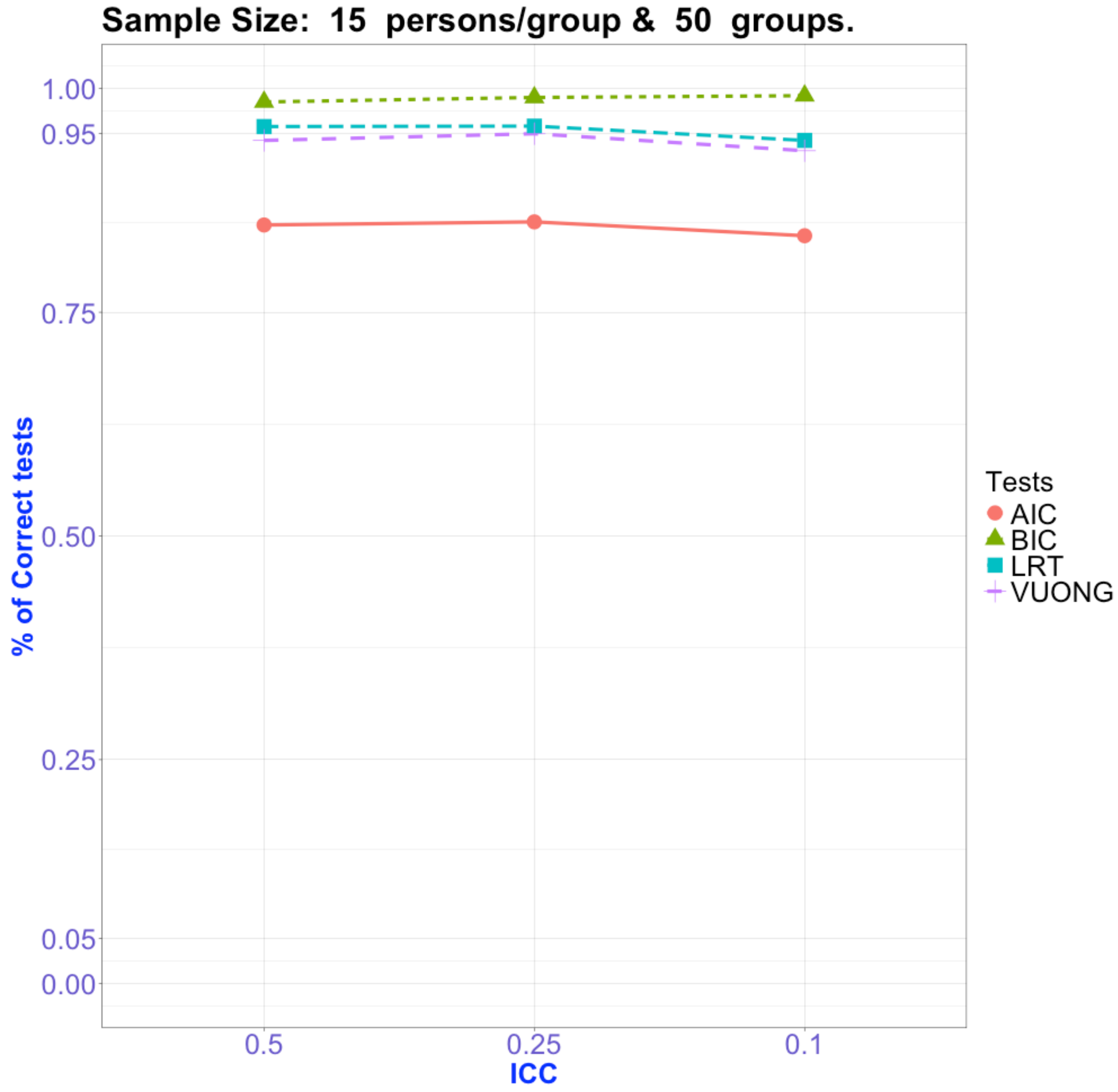


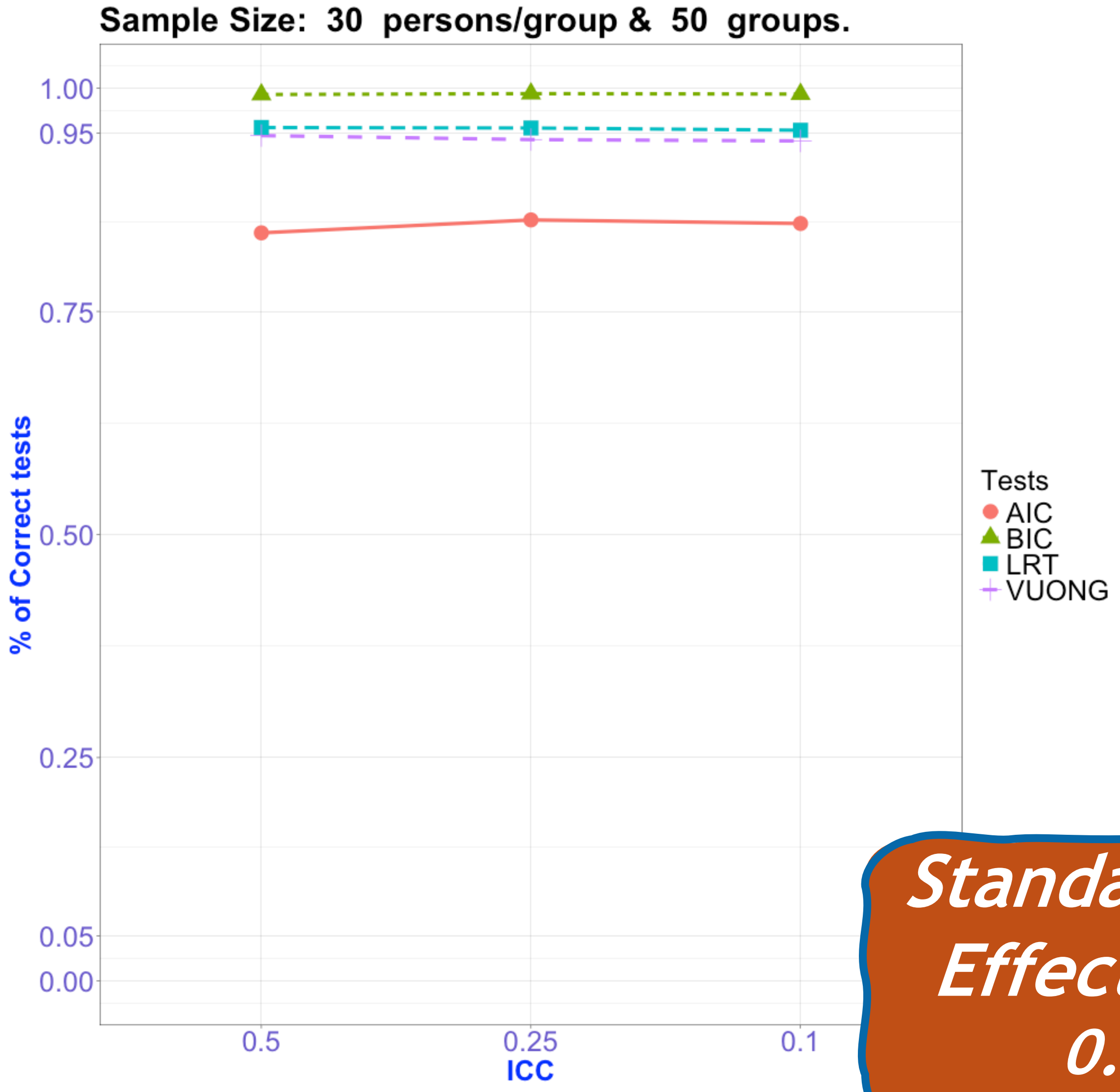


MODEL 0 VS MODEL 1

Standardized Effect of X^1 0

m1 vs m2





MODEL 1 VS MODEL 2

Standardized Effect of X^1 0.25

Simulation MLM with random slopes

Level 1 Time N1	Level 2 Persons N2	Time effect	Variance Inter	Variance Slopes	Residual error
0	25	5	81	0	5
1	50			36	7
2	75				9
	100				11
					13
					15
					25
					50
					75

Simulation MLM with random slopes

$$Y_{ij} = \beta_{00} + \beta_1 \cdot T_{ij} + U_j + e_{ij}$$

$$U_j \sim \mathrm{N}(0, \tau_{00}^2) \quad e_{ij} \sim \mathrm{N}(0, \sigma^2)$$

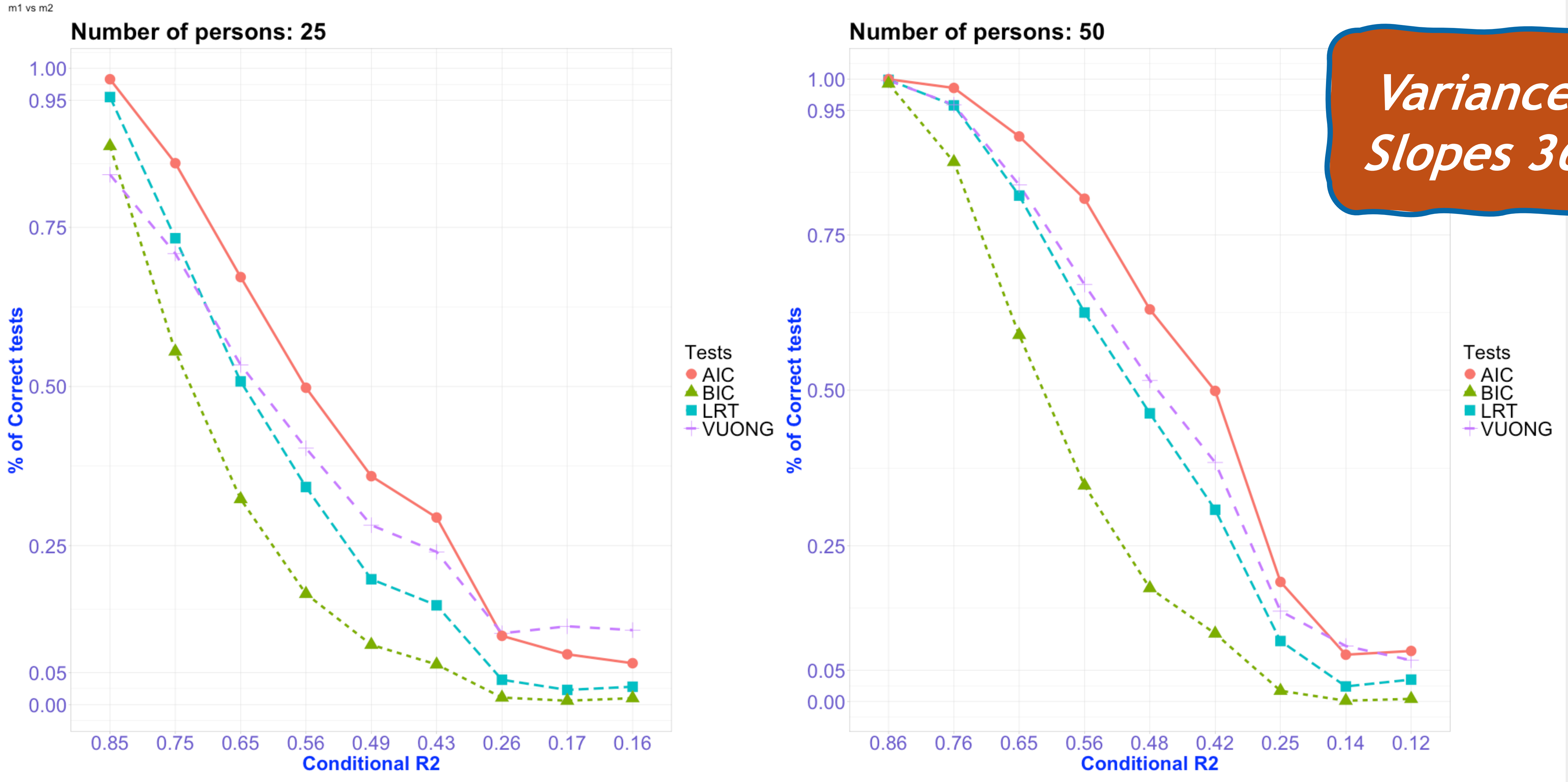
MODEL 1

$$Y_{ij} = \beta_{00} + \beta_1 \cdot T_{ij} + \cdot V_j \cdot T_{ij} + U_j + e_{ij}$$

$$U_j \sim \mathrm{N}(0, \tau_{00}^2) \quad V_j \sim \mathrm{N}(0, \tau_{11}^2) \quad e_{ij} \sim \mathrm{N}(0, \sigma^2)$$

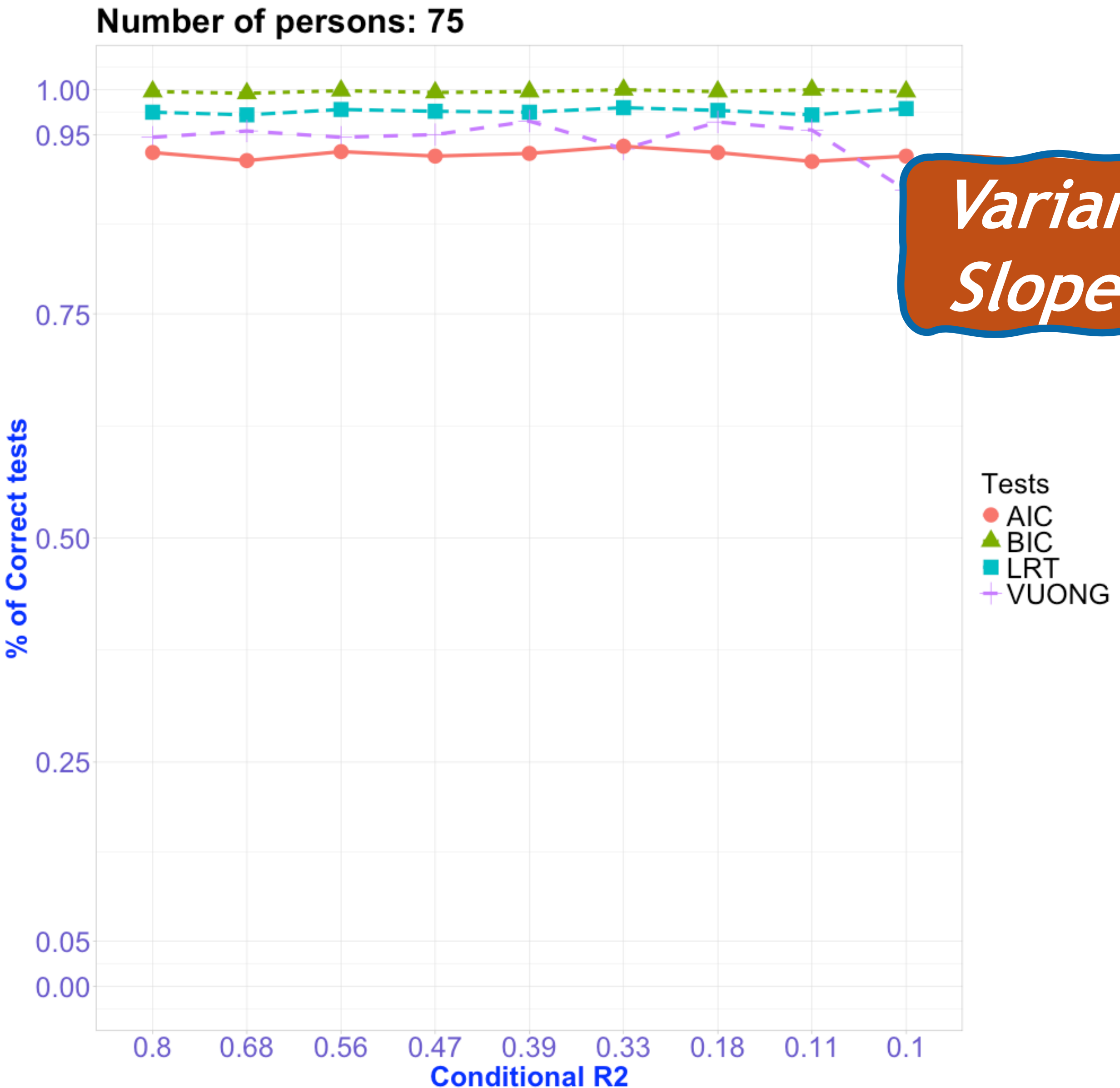
MODEL 2

a

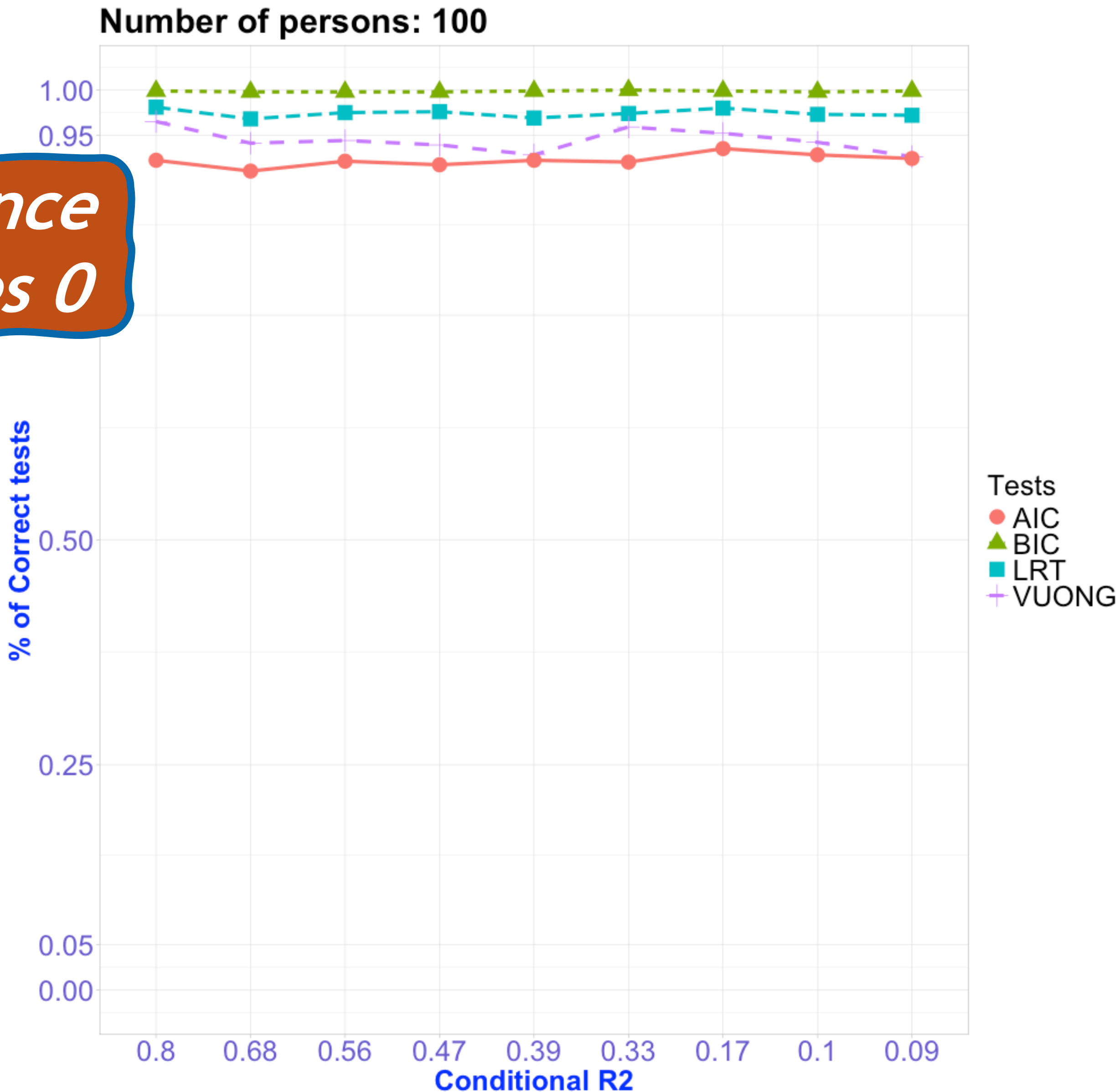


Variance Slopes 36

m1 vs m2



Variance Slopes 0



In nested MLM, if we set the total variance, Vuong test as implemented in nonnest2 perform like LRT, slightly more liberal, it performs a Little bit better when there is lack of power,(N small).

In nested MLM with random slopes, if we set intercept and slopes variances, Vuong test perform as LRT, slightly more liberal and it performs a little bit better than LRT when both loose power, this means Conditional R^2 decreases.

Vuong test can be applied in nested MLM as LRT with similar performance.

Besides, Vuong test can be used to compare non nested models

Thanks for your attention

jsantaola8@alumno.uned.es
jamartinez@psi.uned.es
bfcastilla@psi.uned.es