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Exploring Heterogeneity in Temporal Dynamics With Different Extensions of Time-Varying Coefficient Models

Esther Ulitzsch, Oliver Lüdtke, Steffen Nestler, Therese Snuggerud, & Sverre Johnson

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Content

1. Study I: Motivating Example Introducing TVCMs

2. Study II: Studying Heterogeneity in Individual Temporal Dynamics

Study I: Motivating Example Introducing TVCMs

Snuggerud, T., Ulitzsch, E., Lüdtke, O., Nestler, S., Ebrahimi, O., Vrabel, K., Hoffart, A., Nordahl, H., & Johnson, S.

Motivating Example

Context: Intensive therapy targeting nervousness and threat monitoring as a maladaptive emotion and metacognitive strategy in anxiety patients

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Analysis Approach

$$\begin{bmatrix} x_{it} \\ y_{it} \end{bmatrix} \sim \mathcal{N}_2 \left(\begin{bmatrix} \mu_x(t) \\ \mu_y(t) \end{bmatrix}, \begin{bmatrix} \sigma_x^2(t) & \rho_{xy}(t)\sigma_x(t)\sigma_y(t) \\ \rho_{xy}(t)\sigma_x(t)\sigma_y(t) & \sigma_y^2(t) \end{bmatrix} \right)$$

Approximating Coefficient Functions With Truncated Power Splines

$$\mu_x(t) = \underbrace{a_{x0} + a_{x1}t + a_{x2}t^2}_{\text{quadratic base function}} + \underbrace{\sum_{k=1}^{K} a_{x,2+k}(t-\tau_k)_+^2}_{\text{truncated power functions}}$$

where $(t-\tau_k)_+^2 = \begin{cases} 0 & \text{if } t \leq \tau_k \\ (t-\tau_k)^2 & \text{if } t > \tau_k \end{cases}$

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Here: $K = 5$ knots τ_k, equally spaced over the interval $[l = 0, u = 1]$, with l and u giving the start and end of data collection two weeks before, respectively, after intensive therapy

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$$\sigma_x(t) = \exp\left(b_{x0} + b_{x1}t + b_{x2}t^2 + \sum_{k=1}^{K} b_{x,2+k}(t-\tau_k)_+^2\right)$$

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$$\sigma_x(t) = \exp\left(b_{x0} + b_{x1}t + b_{x2}t^2 + \sum_{k=1}^{K} b_{x,2+k}(t-\tau_k)_+^2\right)$$

$$\rho_{xy}(t) = \tanh\left(c_0 + c_1 t + c_2 t^2 + \sum_{k=1}^{K} c_{2+k}(t-\tau_k)_+^2\right)$$

$$\text{where } (t-\tau_k)_+^2 = \begin{cases} 0 & \text{if } t \leq \tau_k \\ (t-\tau_k)^2 & \text{if } t > \tau_k \end{cases}$$

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Motivating Example: Data

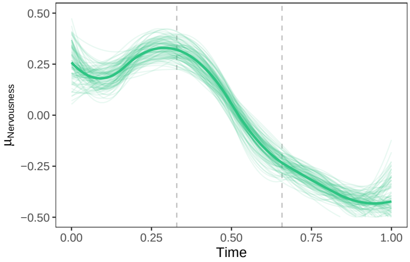
Ecological Momentary Assessment Data

$N = 16$ patients reporting on nervousness and threat monitoring 4 times per day for 6 weeks

2 weeks before therapy | 2 weeks intensive therapy | 2 weeks after therapy

$\rightarrow$ up to 172 measurements per patient

Motivating Example: Results



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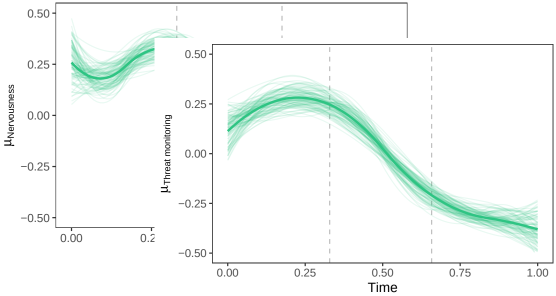

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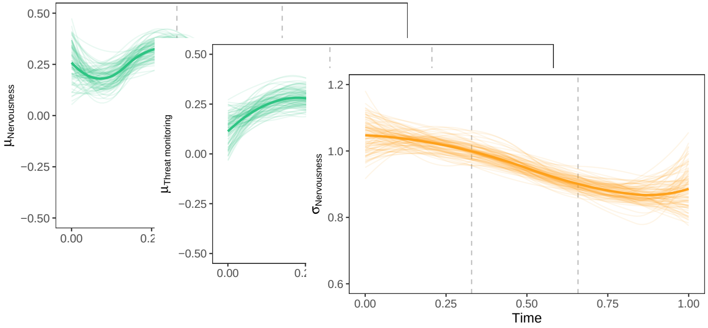

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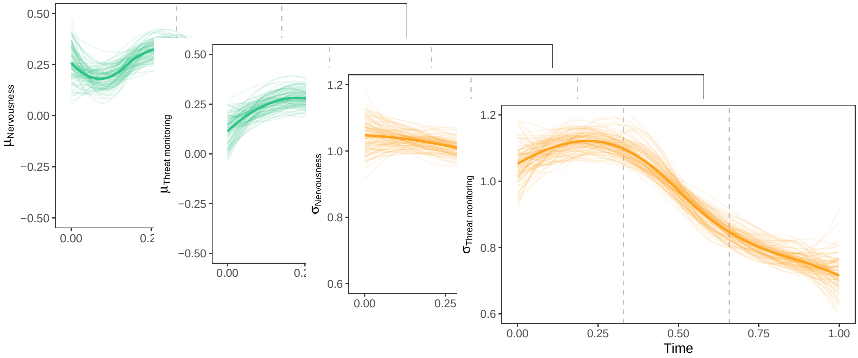

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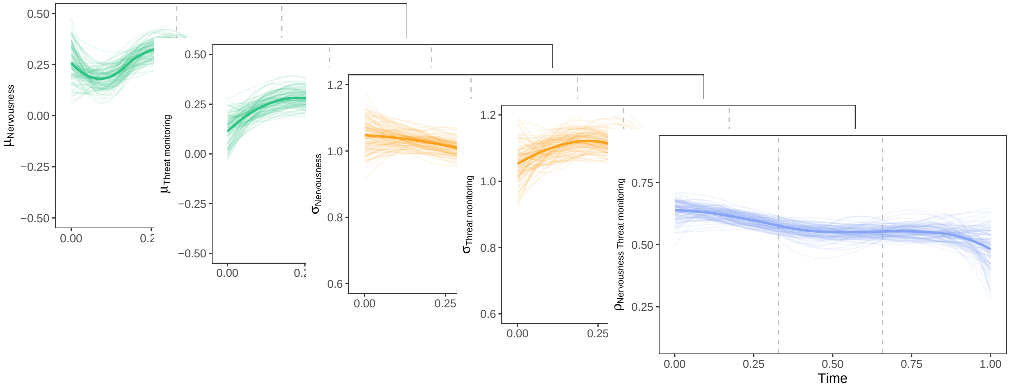

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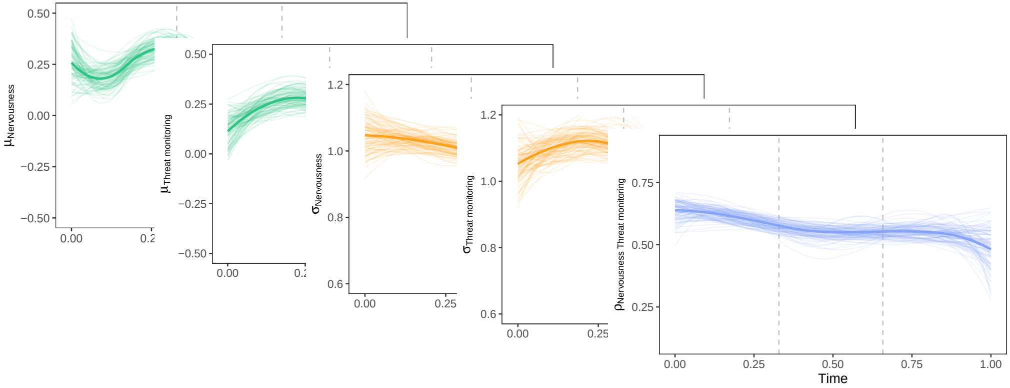

TVCMs aid in uncovering complex patterns of change.

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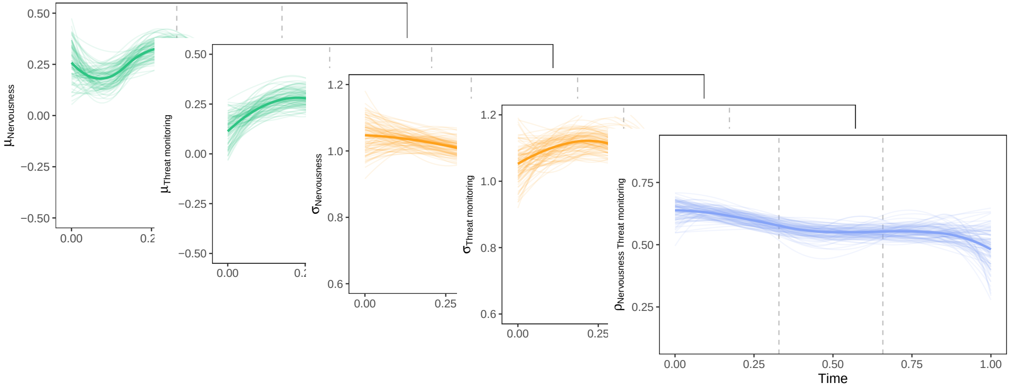

TVCMs aid in uncovering complex patterns of change. However, they can only model the average-level process for the fixed, focal effect (Piccirillo & Foster, 2023).

Study II: Studying Heterogeneity in Individual Temporal Dynamics

Ulitzsch, E., Lüdtke, O., Nestler, S., Snuggerud, T., & Johnson, S.

Objectives

Implementing and exploring multilevel extensions of TVCMs that allow to...

1. ... uncover and describe between-person heterogeneity in *individual-level* temporal dynamics
2. ... explain this heterogeneity with between-person covariates
3. ... explore predictive value of this heterogeneity

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Extended research questions

1. Do temporal dynamics in mean levels, volatility, and coupling of nervousness and threat monitoring differ across individuals and, if so, how?
2. Can differences in temporal patterns be explained by anxiety levels at baseline?
3. Can differences in temporal patterns explain changes in anxiety levels from baseline to three-month follow-up?

Objective I | Option I: Random Intercepts

Individuals may differ in the initial strength of the parameter of interest at the very beginning of the EMA, but trajectories across time parallel each other

$$\mu_{i,x}(t) = a_{i,x0} + a_{x1}t + a_{x2}t^2 + \sum_{k=1}^{K} a_{x,2+k}(t - \tau_k)_+^2$$

$$a_{i,x0} \sim \mathcal{N}(\mu_{ax0}, \phi_{ax0})$$

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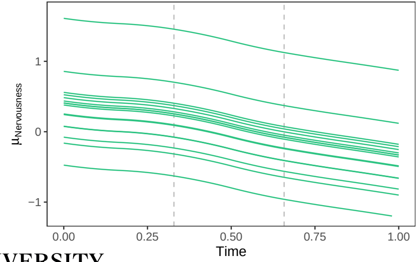

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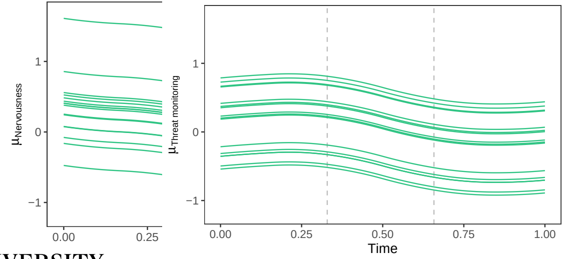

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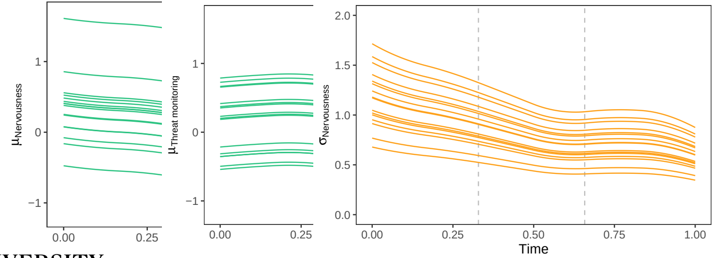

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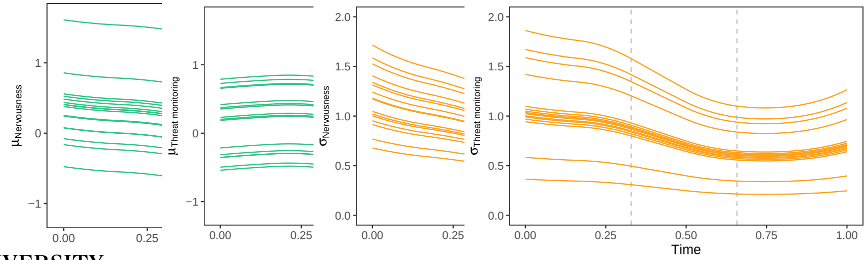

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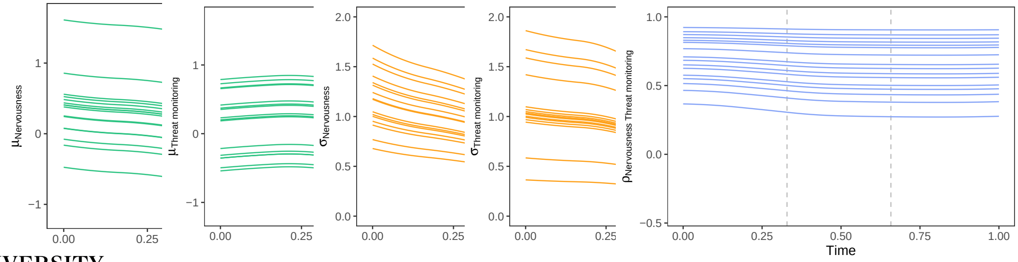

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Allows for all parameters constituting the coefficient functions to be person-specific; functions are smoothed out via partial pooling

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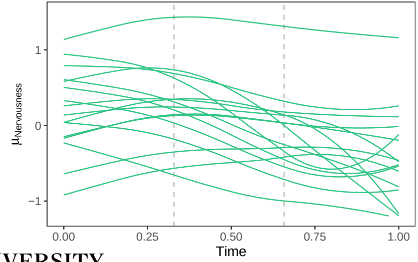

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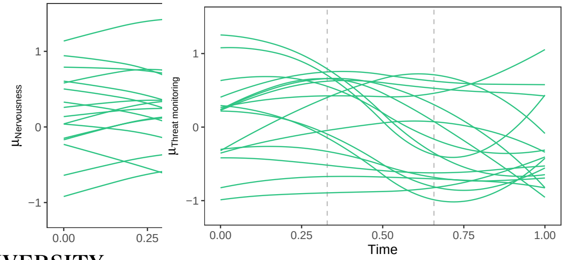

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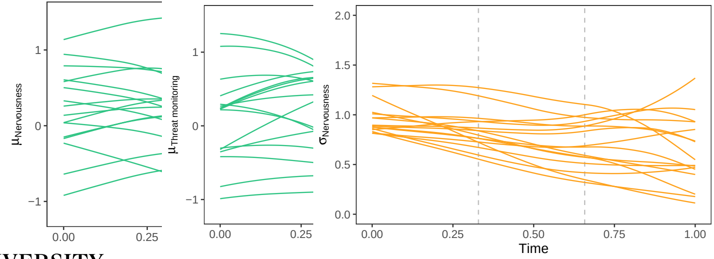

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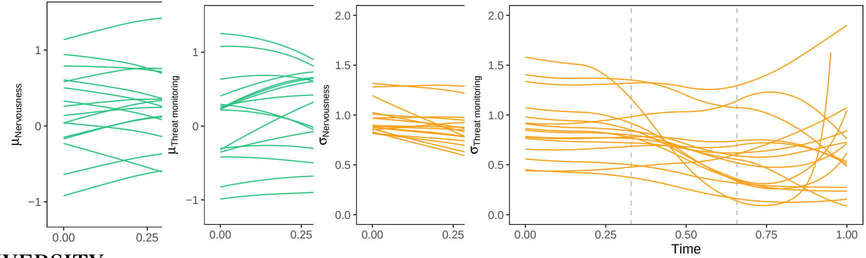

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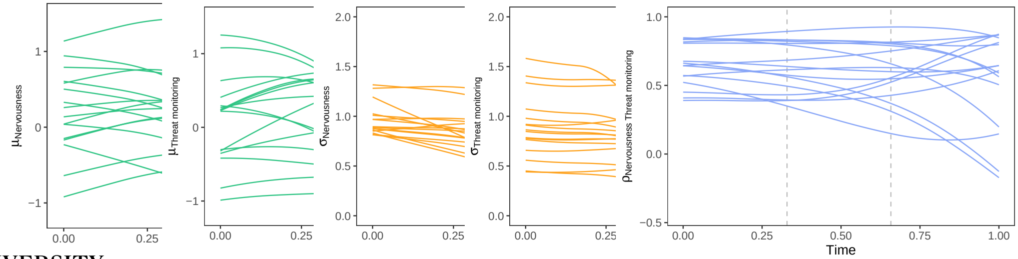

Objective II: Explaining Heterogeneity in Time-Varying Coefficients

$$\mu_{i,x}(t) = a_{i,x0} + a_{i,x1}t + a_{i,x2}t^2 + \sum_{k=1}^{K} a_{i,x,2+k}(t-\tau_k)_+^2$$

$$a_{i,x0} \sim \mathcal{N}(\beta_{0,ax0} + \beta_{1,ax0} \cdot \text{pre}_i, \phi_{ax0}) \quad a_{i,x1} \sim \mathcal{N}(\beta_{0,ax1} + \beta_{1,ax1} \cdot \text{pre}_i, \phi_{ax1})$$

$$a_{i,x2} \sim \mathcal{N}(\beta_{0,ax2} + \beta_{1,ax1} \cdot \text{pre}_i, \phi_{ax2})$$

$$a_{i,x,2+k} \sim \mathcal{N}(\beta_{0,ax,2+k} + \beta_{1,ax,2+k} \cdot \text{pre}_i, \omega_{ax,2+k}) \quad \text{for } k = 1, 2, \ldots, K.$$

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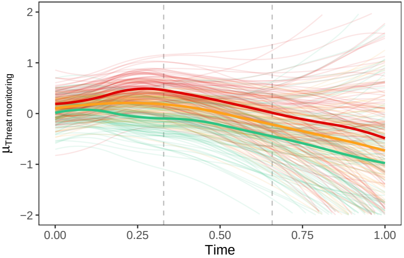
Model-implied trajectories for patients with low to high anxiety levels at baseline



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Objective III: Using Time-Varying Coefficients as Predictors

$$\text{post}_i \sim \mathcal{N}(\gamma_0 + \gamma_p \cdot \text{pre}_i, \xi)$$

$$\text{post}_i \sim \mathcal{N}(\gamma_0 + \gamma_p \cdot \text{pre}_i + \gamma_{c0} \cdot c_{i,0}, \xi)$$

$$\text{post}_i \sim \mathcal{N}(\gamma_0 + \gamma_p \cdot \text{pre}_i + \sum_{k=0}^{K+2} \gamma_{c,k} \cdot c_{i,k}, \xi)$$

Regularized regression model	R^2
Pre-intervention score	.19
Pre-intervention score + random intercept correlation	.57
Pre-intervention score + random coefficients correlation	.64

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Discussion

Summary

- Bringing together time-varying coefficient and multilevel modeling allows to uncover complex patterns of between-person differences in individual-level temporal dynamics

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- In the context of research on therapeutic interventions, patterns of how change unfolds (e.g., how emotions are "uncoupled") may
  - provide insights into the mechanisms underlying these interventions
  - hold diagnostic value (e.g., to predict therapeutic success)

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Major Challenges

- Specifying individual-specific coefficient functions: Balancing model complexity and interpretability

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  - hold diagnostic value (e.g., to predict therapeutic success)

Major Challenges

- Specifying individual-specific coefficient functions: Balancing model complexity and interpretability
- Investigating how individual-specific coefficient functions relate to covariates: Models get quite complicated quickly, and, hence, require large samples to be informative

Thank you for your attention!

Questions? Comments?

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