# Predicting Intersectional Inequalities Using Multilevel Analysis of Individual Heterogeneity and Discriminatory Accuracy (MAIHDA)

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### Two questions for the audience!

- 1. Do you think MAIHDA is a valid application of multilevel modelling?
  - From a theoretical perspective things look a little peculiar!
  - But, from a pragmatic perspective MAIHDA appears useful.

- 2. What is the best way to demonstrate the greater predictive accuracy of multilevel model predicted intersection means over simple means?
  - I have plotted analytic expressions.
  - But, I could have conducted a simulation study.

### Intersectional inequalities

 Traditional studies of inequalities map mean outcomes across the categories of one sociodemographic characteristic (e.g., gender, ethnicity, SES) at a time

| <ul><li>Male</li></ul>   | OR       | White | OR | Low-SES         |
|--------------------------|----------|-------|----|-----------------|
| <ul><li>Female</li></ul> | - Female |       |    | <b>High-SES</b> |

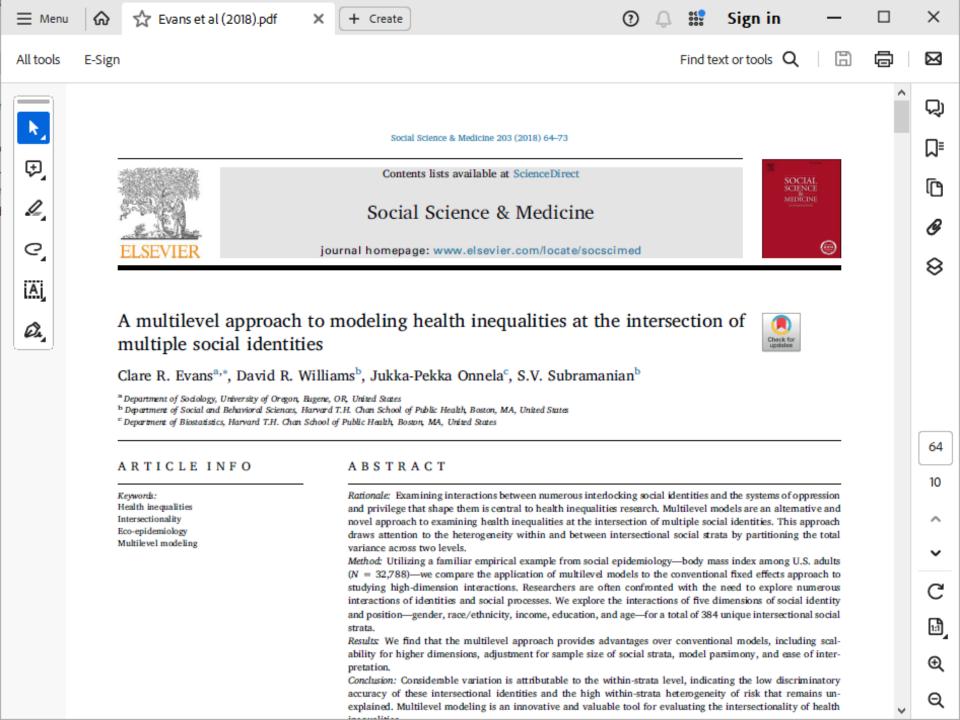
- Intersectional studies of inequalities map mean outcomes across combinations of categories of multiple sociodemographic characteristics.
  - Male, White, Low-SES
  - Male, White, Mid-SES
  - ...
  - Female, Black, High-SES
- Motivated by intersectionality theory:
  - An interest lies in multiple disadvantage and whether it is non-additive.

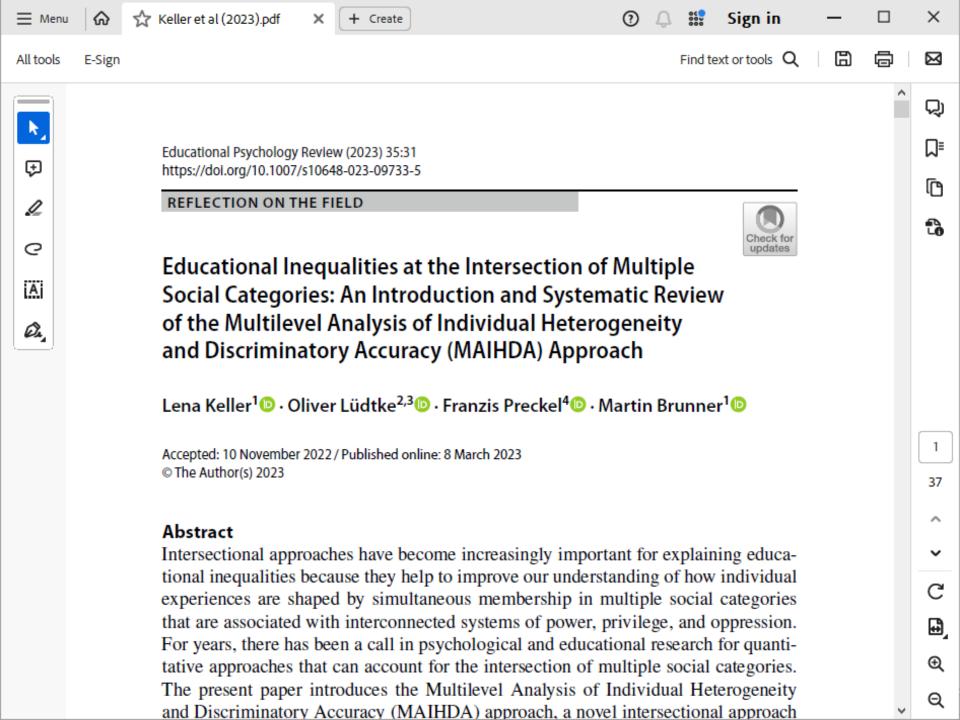
### Simple approach

- The simple approach is to:
  - Calculate the simple mean outcome for each intersection.
  - If we also want to study additivity, we could estimate these means via a saturated linear regression on the multiple characteristics and all twoway and higher-order interactions.
- Argued limitations include:
  - Erratic estimates due to small sample sizes for rarer combinations.
  - Overfitting due to small sample sizes for rarer combinations.
  - Multiple comparisons problem due to many intersections.

### MAIHDA approach

- The MAIHDA approach assumes individuals are nested with intersections and then fits two multilevel models:
  - Model 1: Empty model.
  - Model 2: Includes the sociodemographic characteristics as main effects.
- The intersection means are then predicted post-estimation:
  - Empirical Bayes prediction if estimation by frequentist methods (REML).
  - Posterior means if estimation is by Bayesian methods (MCMC).
- These predicted means are argued to be more accurate than simple means due to shrinkage (partial pooling).





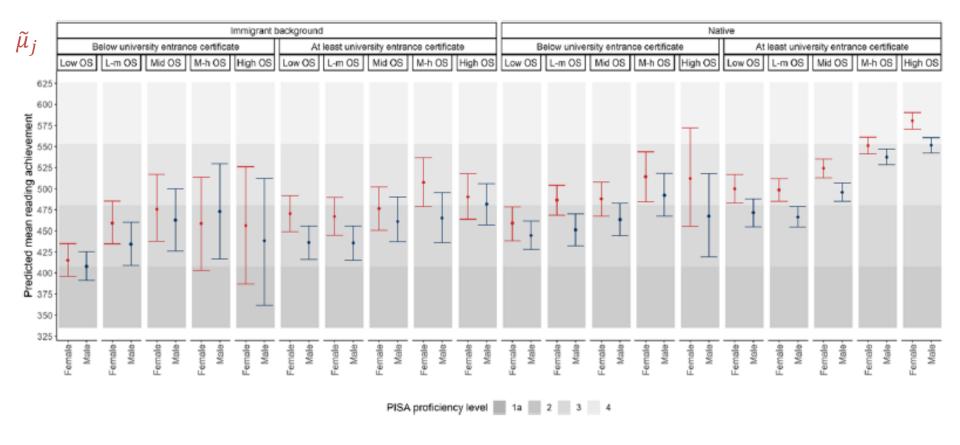
### Key information

- **Data:** PISA 2018 German subsample
- **Outcome:** Student reading achievement at age 15
- **Level 2:** 40 intersections formed from 4 sociodemog. chars. (=  $2 \times 2 \times 2 \times 5$ ):
  - Gender: 2 categories (Male, Female)
  - Immigrant: 2 categories (Native, Immigrant)
  - Education: 2 categories (High-school, University)
  - Occupation: 5 categories (Low, Low-Middle, Middle, Middle-High, High)
- **Level 1:** 5,451 students

Table 3 Parameter estimates for the multilevel models of reading achievement in 15-year-old students

|  | Simple intersectional<br>model estimates<br>[95% CI] | Intersectional interaction<br>model estimates<br>[95% CI] |   |  |
|--|--|---|---|--|
| Fixed effects  |  |   | <ul> <li>MAIHDA Model 1</li> </ul>        |  |
| Intercept  | 478.26<br>[464.27, 492.26]                           | 468.40<br>[452.81, 483.34]                                |   |  |
| Gender   |  |   | A 1 1                                     |  |
| Female (reference)   |  | _   | <ul> <li>An empty two-level</li> </ul>    |  |
| Male   |  | -25.62<br>[-36.46,-14.91]                                 | regression                                |  |
| Immigrant background   |  |   |   |  |
| Native (reference)   |  |   |   |  |
| Immigrant background   |  | -42.46<br>[-53.71,-30.90]                                 | • $\widehat{\text{VPC}} = 0.16$           |  |
| Highest parental education                                   |  |   |   |  |
| Below university entrance certificate (reference)            |  |   | • 16% of the variation                    |  |
| At least university entrance certificate                     |  | 29.79<br>[18.23, 41.99]                                   | in reading scores is                      |  |
| Highest parental occupational status (HISEI)                 |  |   | between intersections                     |  |
| Low occupational status (reference)                          |  |   | between intersections                     |  |
| Low to middle occupational status                            |  | 10.06<br>[-5.85, 26.77]                                   |   |  |
| Middle occupational status                                   |  | 27.63<br>[11.43, 44.58]                                   | • 84% of the variation                    |  |
| Middle to high occupational status  High occupational status |  | 53.09<br>[35.19, 70.36]                                   | in reading scores is within intersections |  |
|  |  | 62.59<br>[40.50, 81.23]                                   | within intersections                      |  |
| Measures of variance   |  |   |   |  |
| Between-stratum variance                                     | 1698.264   | 144.425   |   |  |
| Within-stratum variance                                      | 9011.705   | 9021.200  |   |  |
| VPC  | 15.86%   | 1.64%   |   |  |
| PCV  |  | 91.15%  |   |  |

Note. 95% CI = 95% credible intervals; VPC = variance partition coefficient; PCV = proportional change in the between-strata variance

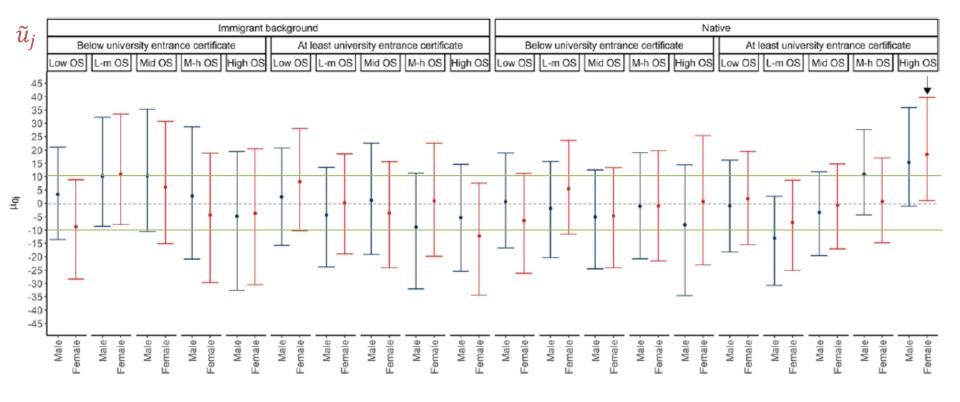


- The predicted intersection means vary greatly (range = 150 points or 1.5 SD).
- The differential patterns suggests the presence of interactions:
  - Gender gap varies by combinations of other sociodemographic chars.
  - Occupation gradient varies by combinations of other sociodemog. chars.

Table 3 Parameter estimates for the multilevel models of reading achievement in 15-year-old students

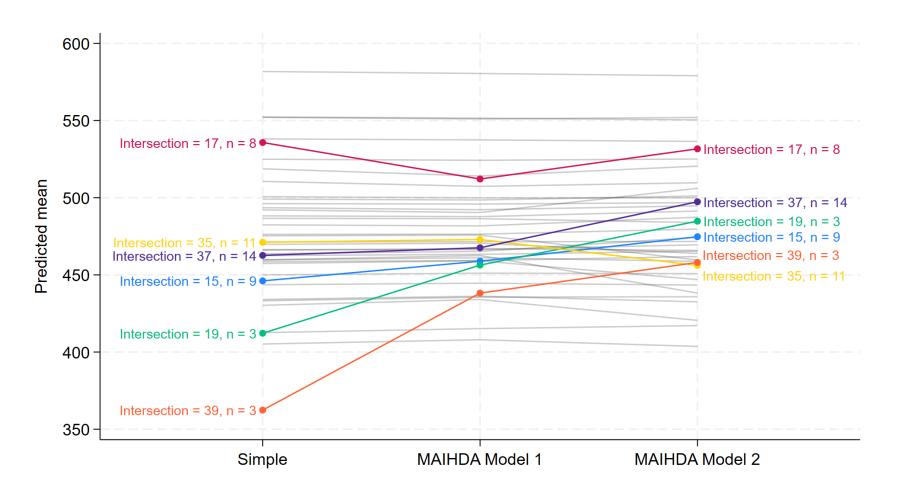
|  | Simple intersectional<br>model estimates<br>[95% CI] | Intersectional interaction<br>model estimates<br>[95% CI] |   |
|--|--|---|---|
| Fixed effects  |  |   |   |
| Intercept  | 478.26<br>[464.27, 492.26]                           | 468.40<br>[452.81, 483.34]                                | <ul> <li>MAIHDA Model 2</li> </ul>        |
| Gender   |  |   |   |
| Female (reference)   |  | -   |   |
| Male   |  | -25.62<br>[-36.46,-14.91]                                 | <ul> <li>Adds main effects of</li> </ul>  |
| Immigrant background   |  |   | the four chars.                           |
| Native (reference)   |  | -   |   |
| Immigrant background   |  | -42.46<br>[-53.71,-30.90]                                 | • $\widehat{PCV} = 0.91$                  |
| Highest parental education   |  |   | $\bullet$ PCV $= 0.91$                    |
| Below university entrance certificate (reference)                                  |  | -   |   |
| At least university entrance certificate   |  | 29.79<br>[18.23, 41.99]                                   | <ul> <li>91% of intersectional</li> </ul> |
| Highest parental occupational status (HISEI)                                       |  |   | inequalities are due to                   |
| Low occupational status (reference)  Low to middle occupational status             |  | 10.06   | •   |
| Low to initiale occupational status  |  | [-5.85, 26.77]  | additive effects                          |
| Middle occupational status   |  | 27.63<br>[11.43, 44.58]                                   |   |
| Middle to high occupational status   |  | 53.09<br>[35.19, 70.36]                                   | • 9% to interactions                      |
| High occupational status   |  | 62.59<br>[40.50, 81.23]                                   | (two-way and higher)                      |
| Measures of variance   |  |   |   |
| Between-stratum variance   | 1698.264   | 144.425   | <ul> <li>Collectively, these</li> </ul>   |
| Within-stratum variance  | 9011.705   | 9021.200  | •   |
| VPC  | 15.86%   | 1.64%   | interactions are                          |
| PCV  |  | 91.15%  | statistically                             |
| Note. 95% CI = 95% credible intervals; VPC = change in the between-strata variance | variance partition coeffic                           | cient; PCV = proportional                                 | significant.                              |

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- Some of the intersection specific mean deviations from additivity are large:
  - Six are 10 points or higher (0.1 of a SD).
- However, only one of these mean deviations is statistically significant:
  - Native, female students with university educated, high occupation parents

### But .... Simple, MAIHDA Model 1 and MAIHDA Model 2 means all differ!

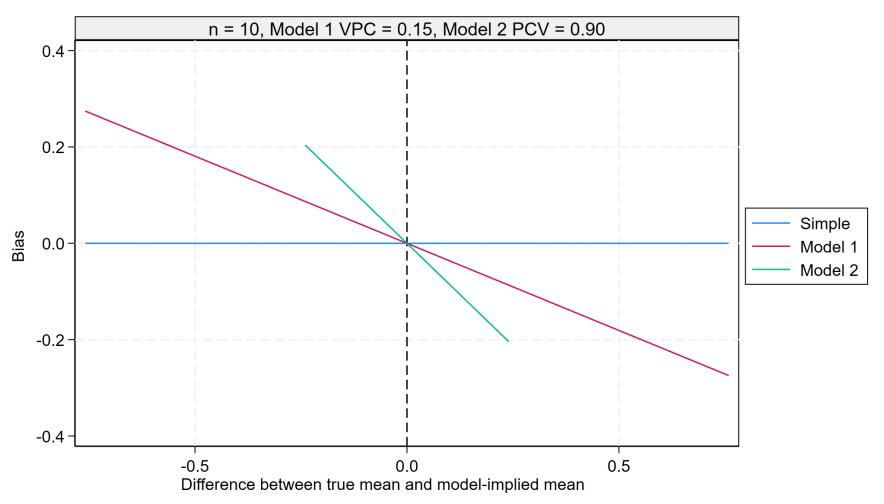


- Differences especially large for small intersections.
- Why do we see differences, and which means are therefore best to report?

## Analytic expressions for Bias, Variance, and MSE for a given intersection mean

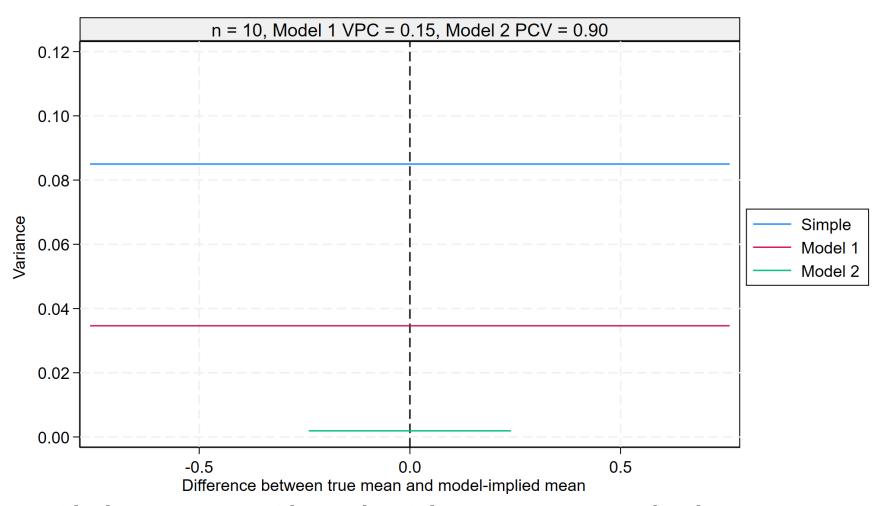
- First, we derive analytic expressions for the ...
  - Bias of each mean
  - Variance of each mean
  - MSE of each mean (= Variance + bias<sup>2</sup>)
- Then, we plot these expressions against the deviation of the true means from the model-implied means (i.e., fixed part) assuming...
  - Model 1 VPC = 15%: The importance of the inequalities
  - Model 2 PCV = 90%: The degree to which inequalities are additive
  - $-n_i = 10$ : Intersection size
- To derive all expressions, we must declare a DGP:
  - We assume Model 2 is the true model

### Bias for a specific intersection



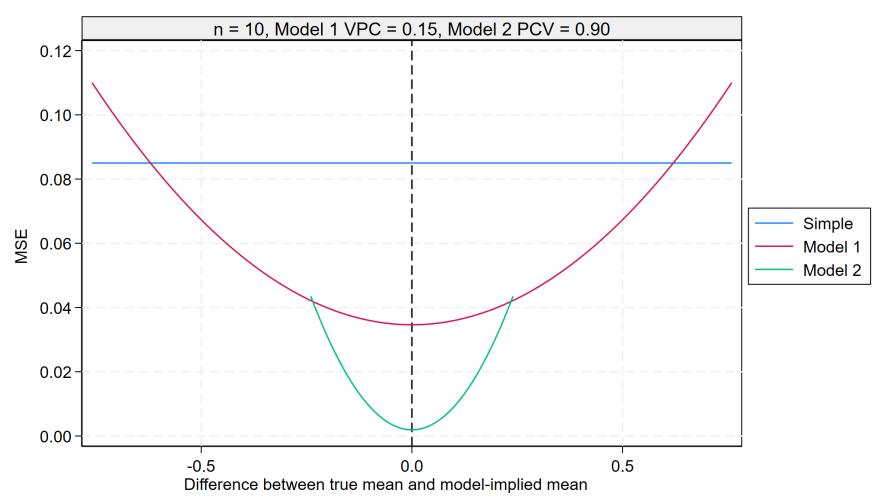
S is unbiased. M1 biased towards grand mean of 0 as shrinkage is towards grand mean. M2 less biased as shrinkage is towards model-implied mean (additive).

### Variance for a specific intersection



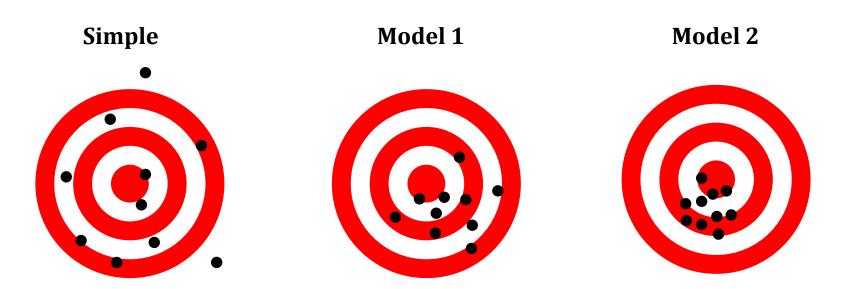
• S highest variance. M1 lower than S due to consistency in shrinkage across samples. M2 lower still due to greater shrinkage induced by high PCV.

### MSE for a specific intersection

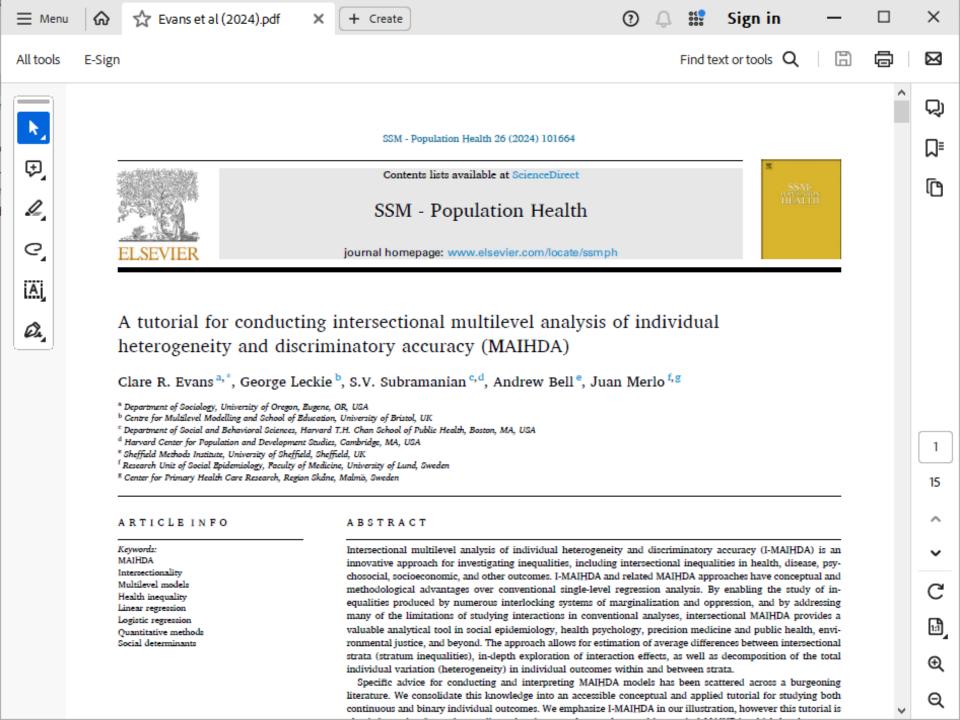


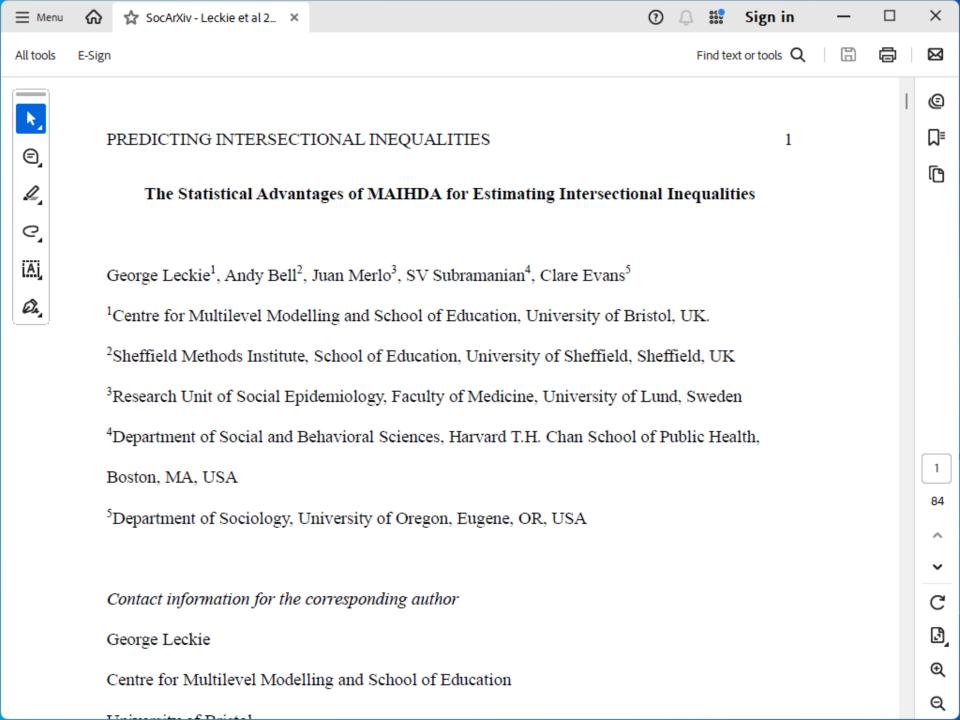
S constant MSE as unbiased and var. constant. All other predictors quadratic. M2 lowest MSE.

### Summary: The bias-variance trade-off



- Across repeated samples ...
  - The simple mean will on average hit bull's eye (unbiased), but in any given sample it may miss the target entirely (large variance).
  - The Model 1 mean will on average miss the bull's eye (biased), but in any given sample it will not be so far off (smaller variance).
  - The Model 2 mean will on average get closer to the bull's eye (smaller bias) than the Model 1 mean and with more consistency (smallest variance).
- In any real analysis we only have one sample, so we go with Model 2.





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Thank you

### **End of talk**