Prior sensitivity analysis in Bayesian SEM and its application in R

Mauricio Garnier-Villarreal (Vrije Universiteit Amsterdam)
Sonja Winter (University of Missouri)
Sara van Erp (Utrecht University)

Posterior and Bayes Rule/Theorem

- Bayesian estimate is a posterior distribution over parameters $Pr(parameters|data)$.
- We can solve for the posterior distribution $Pr(\theta|y)$, represents the probability for our parameter(s) of interest (θ), given data (y)

$$p(\theta|y) = \frac{p(\theta, y)}{p(y)} = \frac{p(y|\theta)p(\theta)}{p(y)}$$

$$p(\theta|y) \propto p(y|\theta)p(\theta)$$

Priors

- $p(\theta)$ is the "prior distribution"
- Represents your knowledge and level of uncertainty
- Represented as probability distributions
- The inclusion of priors is a strength not a weakness.
- Bayesian inference can implement cumulative scientific progress with the inclusion of previous knowledge into the specification of the prior uncertainty

Priors

- Non informative (diffuse)
- Weakly informative
- Strongly informative
- The different types relate to the amount of uncertainty
- The recommended standard one is weakly informative
- Apologetic Bayesian prefer non informative

Priors

- Have more influence on the posterior for smaller samples
- Consider theory, data, and model characteristics
- Are scale dependent, what is a weakly informative prior in one case might be strong in another
- The "intended" priors might differ from the priors in the model due to model constraints, as opaque priors (Merkle et al. 2023)

Prior sensitivity analysis

- Given that prior specification has the potential to alter obtained estimates
- Assess and report prior impact alongside the final model results being reported for a study
- Never blindly rely on default prior settings in software without having a clear understanding of their impact.

Prior sensitivity analysis

- Allows the researcher to methodically examine the impact of prior settings on final results.
- The researcher will often specify original priors based on desired previous knowledge. - After posteriors are estimated and inferences are described, the researcher can then examine the robustness of results to deviations in the priors specified in the original model (Depaoli 2021)

Prior sensitivity analysis

- Defined hypothesis priors
  - Literature, research design, analitycal model, hypothesis
- Estimate model with these priors
- Identify **competing** priors
  - examine how robust the original results are when the priors are altered
- Estimate model with competing models
- **Compare to the original results**
- The final model results are written to reflect the original model results, as well as the sensitivity analysis results. Comments can be made about how robust (or not) the findings were when priors were altered.

Prior sensitivity analysis

- **Compare to the original results**
- We present our recommendation on steps on how to compare models for prior sensitivity analysis in R with the package *blavaan* (Merkle et al. 2021)

Overall model fit

- Approximate fit indices, like CFI, $\hat{\Gamma}$ (Garnier-Villarreal and Jorgensen 2020)
- Information criteria (Vehtari, Gelman, and Gabry 2017)
  - Out-of sample predictive accuracy
  - Leave-one-out (LOO) or Widely Applicable (WAIC)

Posterior distribution

- Plot the posterior distribution for the parameters of interest
- Check if there are **large** changes in function of the priors

blavaan example

```
model <- '
  # latent variable definitions
     ind60 =~ x1 + x2 + x3
     dem60 =~ aa*y1 + bb*y2 + cc*y3 + dd*y4
     dem65 =~ aa*y5 + bb*y6 + cc*y7 + dd*y8

  # regressions
    dem60 ~ ind60
    dem65 ~ ind60 + dem60

  # residual correlations
    y1 ~~ y5
    y2 ~~ y4 + y6
    y3 ~~ y7
    y4 ~~ y8
    y6 ~~ y8
'
```

Prior alternatives

- Default

```
        lambda                beta             theta
  "normal(0,10)"    "normal(0,10)" "gamma(1,.5)[sd]"
```

- Weak

```
        lambda                beta             theta
"normal(1.5, 2)" "normal(0.6, 2)" "gamma(1,1)[sd]"
```

- Strong good

```
         lambda                beta             theta
"normal(1.5, .2)" "normal(0.6, .2)"  "gamma(1,1)[sd]"
```

- Strong bad

```
        lambda                beta             theta
 "normal(4, .2)"  "normal(4, .2)" "gamma(1,1)[sd]"
```

Convergence

```
## save in a list
bflist <- list(A_Default=fit_default,
               B_Weak=fit_weak,
               C_Strong_good=fit_strong,
               D_Strong_bad=fit_strong2)

## convergence
sapply(bflist, FUN=function(obj){max(blavInspect(obj, "rhat"), na.rm=T) })
```

```
    A_Default        B_Weak C_Strong_good  D_Strong_bad
     1.004468      1.002829      1.001976      1.000747
```

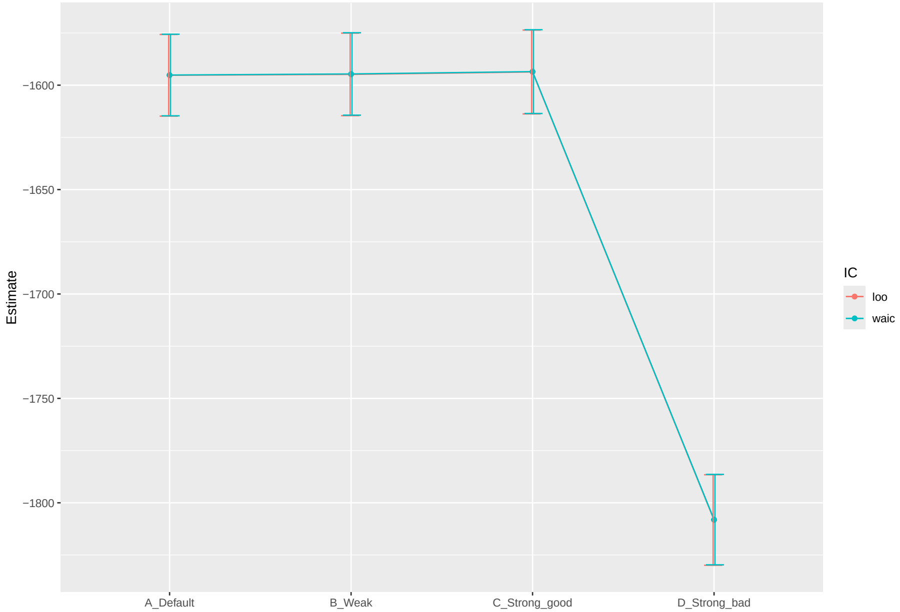
```
## efficiency
sapply(bflist, FUN=function(obj){min(blavInspect(obj, "neff"), na.rm=T) })
```

```
    A_Default        B_Weak C_Strong_good  D_Strong_bad
     1384.290      1456.014      2242.515      2227.667
```

Information Criteria

```
    Estimate       SE     IC          Model
       <num>    <num> <char>         <char>
1: -1595.264 19.55042    loo      A_Default
2: -1595.140 19.54007   waic      A_Default
3: -1594.844 19.76194    loo         B_Weak
4: -1594.628 19.73290   waic         B_Weak
5: -1593.685 20.07740    loo C_Strong_good
6: -1593.505 20.05648   waic C_Strong_good
7: -1808.241 21.67911    loo  D_Strong_bad
8: -1808.046 21.64723   waic  D_Strong_bad
```

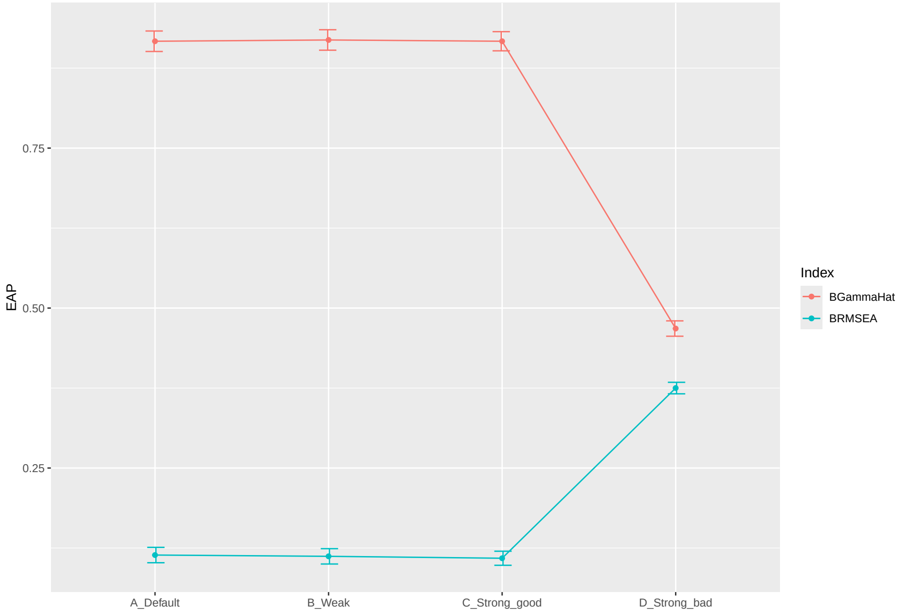
Information Criteria



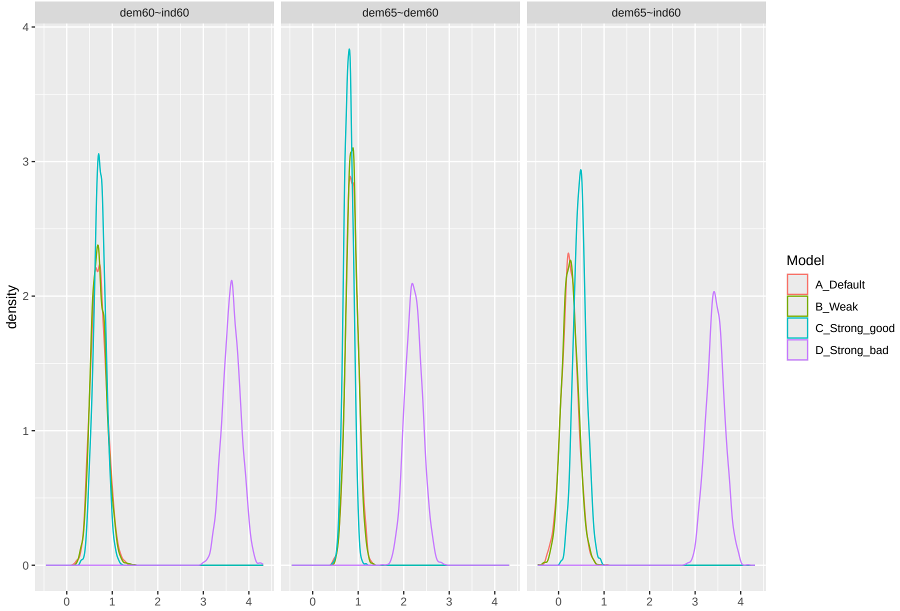
Approximate fit indices

```
      EAP Median   MAP    SD lower upper        Index         Model
    <num>  <num> <num> <num> <num> <num>       <char>        <char>
 1: 0.114  0.113 0.112 0.012 0.094 0.133       BRMSEA     A_Default
 2: 0.917  0.919 0.921 0.016 0.891 0.942    BGammaHat     A_Default
 3: 0.857  0.859 0.864 0.027 0.811 0.899 adjBGammaHat     A_Default
 4: 0.781  0.784 0.790 0.040 0.714 0.843          BMc     A_Default
 5: 0.112  0.112 0.112 0.012 0.093 0.131       BRMSEA        B_Weak
 6: 0.919  0.920 0.921 0.016 0.893 0.943    BGammaHat        B_Weak
 7: 0.860  0.862 0.864 0.027 0.816 0.902 adjBGammaHat        B_Weak
 8: 0.784  0.787 0.789 0.039 0.718 0.846          BMc        B_Weak
 9: 0.109  0.109 0.108 0.011 0.092 0.127       BRMSEA C_Strong_good
10: 0.917  0.918 0.919 0.015 0.892 0.940    BGammaHat C_Strong_good
11: 0.867  0.869 0.871 0.024 0.827 0.905 adjBGammaHat C_Strong_good
12: 0.779  0.782 0.785 0.038 0.716 0.840          BMc C_Strong_good
13: 0.375  0.375 0.375 0.009 0.360 0.390       BRMSEA  D_Strong_bad
14: 0.468  0.468 0.467 0.012 0.449 0.488    BGammaHat  D_Strong_bad
15: 0.211  0.211 0.210 0.018 0.183 0.241 adjBGammaHat  D_Strong_bad
16: 0.044  0.044 0.043 0.007 0.033 0.054          BMc  D_Strong_bad
```

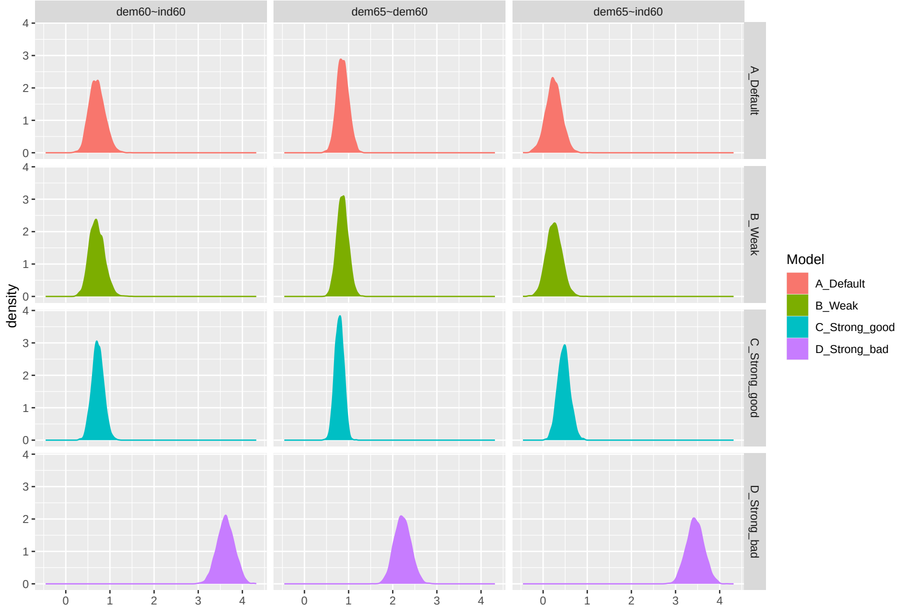
Approximate fit indices



Posterior distributions



Posterior distributions



Caveat

"There is one final caveat related to this issue. It is important to report the results based on the original prior no matter what the sensitivity analysis results convey. In other words, do not modify the original priors because of something that was unveiled in the sensitivity analysis. The practice of modifying the priors based on finding more desirable results within a sensitivity analysis would be considered Bayesian HARKing (hypothesizing after results are known; Kerr, 1998). At the very least, this action would be considered a questionable research practice, but I would argue it is a misleading–or even deceiving–action." (Depaoli 2021)

Conclusions

- We can compare do prior sensitivity analysis by compaing model fit and posterior distribution
- We will create user friendly R functions, and add them to the **semTools** package
- Creating easy ways to do and report the priors sensitivity

Contact

m.garniervillarreal@vu.nl

https://maugavilla.github.io/

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