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Ipsativity indices for forced-choice assessments

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- Ipsativity vs. Normativity
- Normativity Indicator
- Simulation with Real Item Pool
- Conclusions
- Future Directions

Ipsativity:

- From the Latin *ipse* (he, himself): The scores can only be interpreted for oneself, not allowing to compare between persons.
- Interdependence between multidimensional scales derived generally from the data format.
- E.g., forced-choice responses.

The forced-choice format:

- Respondents are required to rank statements (items) in a block:

	Most like me
I adapt to setbacks. (ES)	○
I fulfill my commitments. (CO)	○

- Under CTT, scores reflect the number of endorsements of the items in each dimension.
- The total number of endorsements across all dimensions is constant.
- That is, the scores reflect proportions of the endorsement of different dimensions out of a constant total.

Consequences of ipsative scoring:

A respondent has a higher score in trait A than trait B.

~~Respondent 1 has a higher score than Respondent 2.~~

No absolute interpretation of the scores (underidentified origin).

Some properties of ipsative scores (Clemans, 1966; Hicks, 1970):

- Sum/mean of scores is constant across respondents .
  - A higher endorsement of items in one dimension necessarily implies a lower endorsement of those of the other dimensions.
  - Impossibility to rank respondents in multivariate space.
- Negative interdependence between scales:
  - For truly independent traits, observed correlation of $\frac{-1}{D-1}$.
- Sum/mean of validity coefficients of 0.
- Thus, internal consistency and external validity are compromised.

Overcoming ipsativity:

- Building strategies to avoid constant sum/mean of scores:
  - CTT: including reversely coded items, with negative scoring key.
  - IRT: Using items with diverse discrimination parameters.
- Still, it is difficult to determine the amount of ipsativity in scores.

Normativity Score Decomposition

- Conceptually, scores are fully ipsative if their sum (or mean) is constant across all individuals.
- It is our aim to quantify the true variance of within-person average across dimensions ($\bar{\hat{\theta}}_i$).
- I.e., Variation of within-person mean scores that is due to true within-person mean score variation.

Normativity Score Decomposition

$$\hat{\theta}_{i,k} = \bar{\hat{\theta}}_i + \left(\hat{\theta}_{i,k} - \bar{\hat{\theta}}_i\right)$$

Estimated Score

Within-person Average Estimate

Within-person Differential Score Estimate

$$\bar{\hat{\theta}}_i = \bar{\theta}_i + \varepsilon_{\text{mean}}$$

$$\left(\hat{\theta}_{i,k} - \bar{\hat{\theta}}_i\right) = \left(\theta_{i,k} - \bar{\theta}_i\right) + \varepsilon_{\text{diff}_k}$$

Normativity Score Decomposition

- Estimated score variance can be decomposed as:

$$\mathrm{var}\left(\hat{\theta}_{i,k}\right) = \mathrm{var}\left(\bar{\hat{\theta}}_i\right) + \mathrm{var}\left(\hat{\theta}_{i,k} - \bar{\hat{\theta}}_i\right) + 2 \cdot \mathrm{cov}\left(\bar{\hat{\theta}}_i, \hat{\theta}_{i,k} - \bar{\hat{\theta}}_i\right)$$

- True score variance and error variance can be decomposed in similar fashion.

(Theoretical) Normativity Index:

In terms of asymptotic error $\mathrm{Cov}(\widehat{\boldsymbol{\theta}}_i|\boldsymbol{\theta}_i)$, it can be found that:

- The error variance of the within-person average estimate equals the mean of the conditional asymptotic error covariance matrix.

$$\mathrm{var}\left(\bar{\hat{\theta}}_i\right) = \mathrm{mean}[\mathrm{Cov}(\widehat{\boldsymbol{\theta}}_i|\boldsymbol{\theta}_i)]$$

(Theoretical) Normativity Index:

Reliability (normativity) of mean scores:

$$r^2_{\bar{\hat{\theta}}\bar{\theta}} = \frac{\int \mathrm{var}\left(\bar{\hat{\theta}}\right) g(\bar{\hat{\theta}})}{\mathrm{var}(\bar{\theta})},$$

where $\mathrm{var}(\bar{\theta})$ is the mean of the populational trait covariance matrix, $\mathrm{Cov}(\boldsymbol{\theta})$.

(Empirical) Normativity Index:

For empirical scores:

- The posterior standard deviation (PSD) of within-person means can be computed from square root of the average of the posterior covariance matrix.

(Empirical) Normativity Index:

Reliability (normativity) of empirical mean EAP scores:

$$r^2_{\bar{\hat{\theta}}\bar{\theta}} = \frac{\mathrm{var}\left(\bar{\hat{\theta}}\right)}{\mathrm{var}\left(\bar{\hat{\theta}}\right) + \left[N^{-1}\sum_{n=1}^{N}\mathrm{PSD}^2\left(\bar{\hat{\theta}}\right)\right]},$$

where $\mathrm{var}\left(\bar{\hat{\theta}}\right)$ is the variance of the estimated within-person averages.

Simulation Study with a Real Item Pool

Study Design:

- Block pool: 154 equally-keyed blocks of Big Five factors.
- Test lengths: 10, 20, 30, 40, 50, and 60 blocks.
- 1,000 simulees with $\boldsymbol{\theta} \sim MVN(\mathbf{0}, \Sigma_{NEO\text{-}PI})$.
- Criterion 1: simulated with correlation 0.3 with every $\boldsymbol{\theta}$ (multiple $R^2 = 0.36$). Big common variance with all traits.
- Criterion 2: simulated with correlation 0.6 with θ_1 (multiple $R^2 = 0.36$). Common variance with trait 1 only.

Procedure:

- For each test length condition, randomly assemble 100 tests from the pool (balanced content constraint).

Data Analysis:

- Recovery of true normativity (squared correlation between true and estimated within-person mean).

- Effect of normativity on multiple R-squared with criteria:

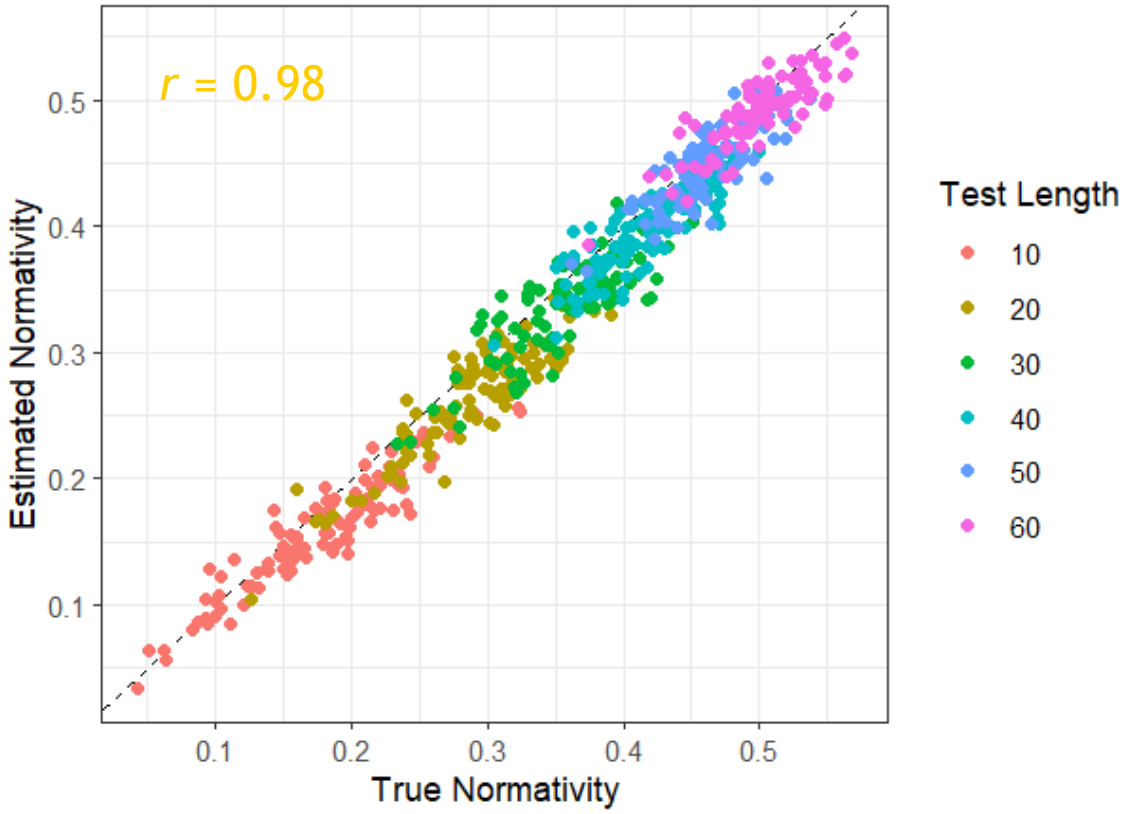
$$Y_1 \sim \hat{\theta}_1 + \hat{\theta}_2 + \hat{\theta}_3 + \hat{\theta}_4 + \hat{\theta}_5$$

$$Y_2 \sim \hat{\theta}_1 + \hat{\theta}_2 + \hat{\theta}_3 + \hat{\theta}_4 + \hat{\theta}_5$$

- Effect of normative on correct selection rate ($\boldsymbol{\theta} > 0$).

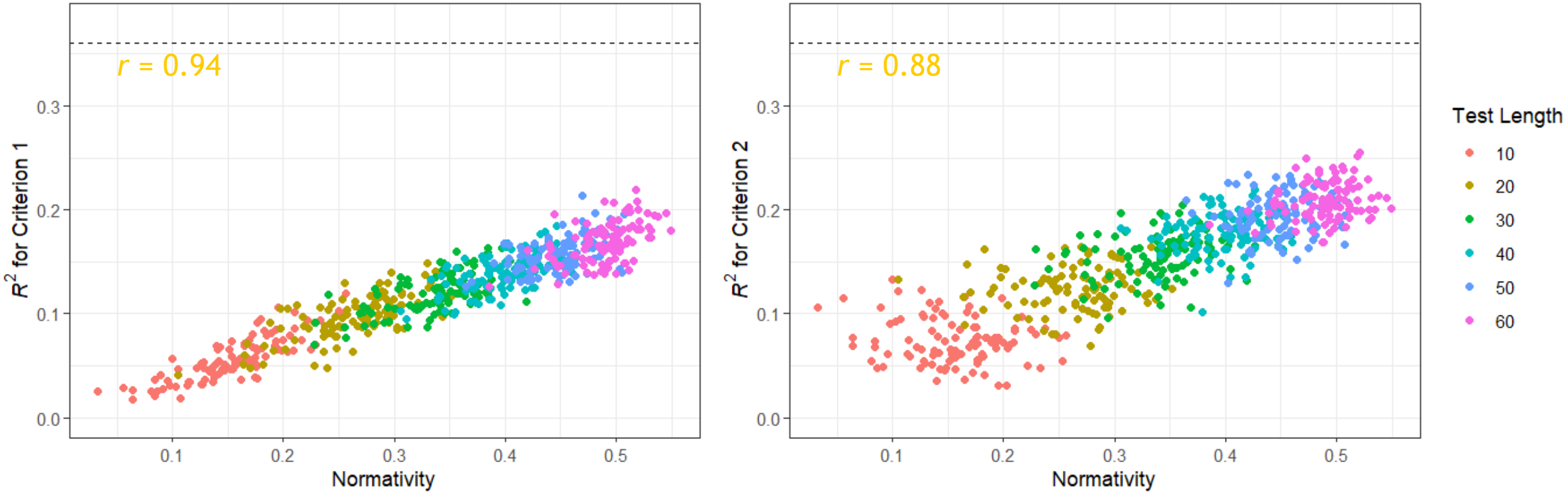
Results:

- Recovery of true normativity:



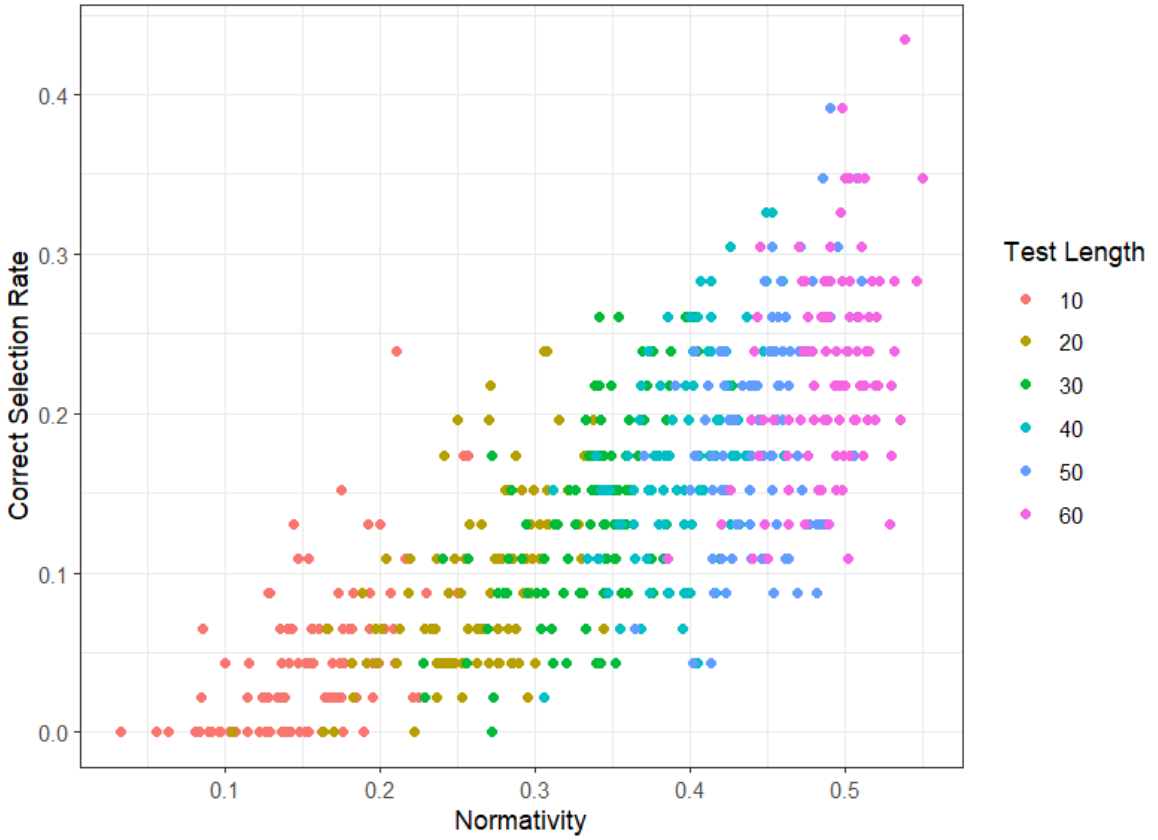
Results:

- Effect of normativity on multiple R-squared with criteria:



Results:

- Effect of normativity on correct selection rate ($\boldsymbol{\theta} > 0$):



Conclusions:

- Calculation of normativity indices offer good recovery of the squared correlation between true and estimated within-person means.
- Normativity has a positive effect on external validity estimates, especially when criteria are related to multiple traits (e.g., life satisfaction).
- Normativity has a positive effect on correct selection rates (r = 0.78), more than reliability alone (r = 0.72).

Thank you!
¡Muchas gracias!

For more information:

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